where $\mathcal{F}$ is Faraday constant (96.487×10^6 C/k mol) of electrons or 96,487 kJ/V-kmol of electrons, n is the number of moles of reactant having a valence j, and V is the cell terminal voltage. The ideal cell terminal voltage is given by:

$$V_i = \frac{\Delta G}{\mathcal{F} n j} \tag{16.39}$$

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

Solved Examples

Example 16.1

Starting with n_0 moles of NH_3, which dissociates according to the equation

$$NH_3 \rightleftharpoons \frac{1}{2} N_2 + \frac{3}{2} H_2,$$

show that at equilibrium

$$K = \frac{\sqrt{27}}{4} \frac{\varepsilon_e^2}{1-\varepsilon_e^2} \cdot p$$

Solution

$$NH_3 \rightleftharpoons \frac{1}{2} N_2 + \frac{3}{2} H_2$$

Comparing the equation with

$$\nu_1 A_1 + \nu_2 A_2 \rightleftharpoons \nu_3 A_3 + \nu_4 A_4$$

we have

$$A_1 = NH_3, \nu_1 = 1, n_1 = n_0 (1 - \varepsilon), x_1 - \frac{1-\varepsilon}{1+\varepsilon}$$

$$A_3 = N_2, \nu_3 = \frac{1}{2}, n_3 = \frac{n_0 \varepsilon}{2}, x_3 = \frac{\varepsilon}{2(1+\varepsilon)}$$

$$A_4 = H_2, n_4 = \frac{3}{2}, v_4 = \frac{3 n_0 \varepsilon}{2}, x_4 = \frac{3\varepsilon}{2(1+\varepsilon)}$$

$$\Sigma n = n_0 (1 + \varepsilon)$$

Substituting in the law of mass action

$$K = \left[\frac{x_3^{\nu_3} \cdot x_4^{\nu_4}}{x_1^{\nu_1} \cdot x_2^{\nu_2}}\right]_{\varepsilon=\varepsilon_e} p^{\nu_3+\nu_4-\nu_1-\nu_2}$$

$$= \frac{\left[\frac{\varepsilon_e}{2(1+\varepsilon_e)}\right]^{1/2} \left[\frac{3\varepsilon_e}{2(1+\varepsilon_e)}\right]^{3/2}}{\left(\frac{1-\varepsilon_e}{1+\varepsilon_e}\right)} \cdot p^{\frac{3}{2}+\frac{1}{2}-1}$$

$$= \frac{\sqrt{27}\, \varepsilon^2}{4(1+\varepsilon_e)^2} \cdot \frac{1+\varepsilon_e}{1-\varepsilon_e} \cdot p = \frac{\sqrt{27}}{4} \cdot \frac{\varepsilon_e^2}{1-\varepsilon_e^2} \cdot p$$

Proved.

Example 16.2

At 35°C and 1 atm. the degree of dissociation of N_2O_4 at equilibrium is 0.27. (a) Calculate K. (b) Calculate ε_e at the same temperature when the pressure is 100 mm Hg. (c) The equilibrium constant for the dissociation of N_2O_4 has the values 0.664 and 0.141 at temperatures 318 and 298 K respectively. Calculate the average heat of reaction within this temperature range.

Solution Nitrogen tetroxide dissociates according to the equation

$$\mathrm{N_2O_4} \rightleftharpoons \mathrm{2NO_2}$$

Comparing this equation with

$$\nu_1 A_1 + \nu_2 A_2 \rightleftharpoons \nu_3 A_3 + \nu_4 A_4$$

$$A_1 = \mathrm{N_2O_4},\ \nu_1 = 1,\ n_1 = n_0(1-\varepsilon),\ x_1 = \frac{1-\varepsilon}{1+\varepsilon}$$

$$A_3 = \mathrm{NO_2},\ \nu_3 = 2,\ n_3 = 2n_0\,\varepsilon,\ x_3 = \frac{2\varepsilon}{1+\varepsilon}$$

$$\Sigma n = n_0(1+\varepsilon)$$

$$\nu_2 = 0,\ \nu_4 = 0$$

Substituting in the law of mass action

$$K = \left[\frac{x_3^{\nu_3}\cdot x_4^{\nu_4}}{x_1^{\nu_1}\cdot x_2^{\nu_2}}\right]_{\varepsilon_e}\cdot p^{\nu_3+\nu_4-\nu_1-\nu_2}$$

$$= \frac{\left(\dfrac{2\varepsilon_e}{1+\varepsilon_e}\right)^2}{\dfrac{1-\varepsilon_e}{1+\varepsilon_e}}\cdot p^{2+0-1+0} = \frac{4\varepsilon_e^2 p}{1-\varepsilon_2^2}$$

(a) When $\varepsilon_e = 0.27$ and $p = 1$ atm.

$$K = \frac{4(0.27)^2\cdot 1}{1-(0.27)^2} = 0.3145 \text{ atm.}$$ *Ans.*

(b) Taking the value of K to be the same at the same temperature

$$0.3145 \text{ atm.} = \frac{4\varepsilon_e^2}{1-\varepsilon_e^2}\cdot\frac{100}{760}$$

$$\therefore \qquad \varepsilon_e = 0.612$$ *Ans*

(c) The heat of reaction is given by

$$\Delta H = -2.30\ \bar{R}\,\frac{\mathrm{d}\log K}{\mathrm{d}\dfrac{1}{T}}$$

$$= 2.30\ \bar{R}\,\frac{T_1T_2}{T_1-T_2}\log\frac{K_1}{K_2}$$

$$= 2.30 \times 8.3143 \times \frac{318\times 298}{318-298}\log\frac{0.664}{0.141}$$

$$= 90700 \log 4.7 = 61{,}000 \text{ kJ/kg mol}$$ *Ans.*

Example 16.3

For the chemical reaction

$$CO_2 + H_2 \rightleftharpoons CO + H_2O$$

the equilibrium value of the degree of reaction at 1200 *K is* 0.56. *Determine the equilibrium constant and the Gibbs function change.*

Solution

$$CO_2 + H_2 \rightleftharpoons CO + H_2O$$

Here, $\nu_1 = \nu_2 = \nu_3 = \nu_4 = 1$

At equilibrium, $n_1 = 1 - \varepsilon, n_2 = 1 - \varepsilon,$

$n_3 = \varepsilon$ and $n_4 = \varepsilon$

$\therefore$ $\sum n = 2$

Therefore, $x_1 = \dfrac{1-\varepsilon}{2} \quad x_2 = \dfrac{1-\varepsilon}{2} \quad x_3 = \dfrac{\varepsilon}{2} \quad x_4 = \dfrac{\varepsilon}{2}$

Substituting $\varepsilon = 0.56,$

$$x_1 = 0.22 \quad x_2 = 0.22 \quad x_3 = 0.28 \quad x_4 = 0.28$$

$$K = \frac{x_3^{\nu_3}\, x_4^{\nu_4}}{x_1^{\nu_1}\, x_2^{\nu_2}} \times p^{\nu_3+\nu_4-\nu_1-\nu_2} = \frac{0.28 \times 0.28}{0.22 \times 0.22} = 1.62$$

$$\Delta G = -\bar{R}T \ \ln K + \bar{R}T \ln p^{(\nu_3+\nu_4-\nu_1-\nu_2)}$$

$$= -\bar{R}T \ \ln K = -8.3143 \times 1200 \ln 1.62 = -4813.2 \text{ J/g mol}$$

Example 16.4

Prove that for a mixture of reacting ideal gases,

$$\frac{d}{d\varepsilon} \ln \frac{x_3^{\nu_3}\, x_4^{\nu_4}}{x_1^{\nu_1}\, x_2^{\nu_2}} = \frac{(n_0 + n_0')}{\Sigma n_k} \frac{(\nu_1 + \nu_2)(\nu_3 + \nu_4)}{\varepsilon(1-\varepsilon)}$$

which is always positive.

Solution From the law of mass action, the equilibrium constant is given by:

$$K = \frac{x_3^{\nu_3}\, x_4^{\nu_4}}{x_1^{\nu_1}\, x_2^{\nu_2}} p^{\nu_3+\nu_4-\nu_1-\nu_2}$$

$$= \frac{[n_3/\Sigma n]^{\nu_3}\, [n_4/\Sigma n]^{\nu_4}}{[n_1/\Sigma n]^{\nu_1}\, [n_2/\Sigma n]^{\nu_2}} p^{\nu_3+\nu_4-\nu_1-\nu_2}$$

$$= \frac{n_3^{\nu_3}\, n_4^{\nu_4}}{n_1^{\nu_1}\, n_2^{\nu_2}} [p/\Sigma n]^{\Delta\nu}$$

where $\Sigma n = n_1 + n_2 + n_3 + n_4$ and $\Delta\nu = \nu_3 + \nu_4 - \nu_1 - \nu_2$.

By logarithmic differentiation,

$$\frac{dK}{K} - \Delta\nu \frac{dp}{p} + \Delta\nu \frac{d\Sigma n}{\Sigma n} = \nu_3 \frac{dn_3}{n_3} + \nu_4 \frac{dn_4}{n_4} - \nu_1 \frac{dn_1}{n_1} - \nu_2 \frac{dn_2}{n_2} \qquad (1)$$

Now,

$$n_1 = (n_0 + n'_0)\, \nu_1 (1 - \varepsilon)$$

$$n_2 = (n_0 + n'_0)\, \nu_2 (1 - \varepsilon)$$

$$n_3 = (n_0 + n'_0)\, \nu_3 \varepsilon$$

$$n_4 = (n_0 + n'_0)\, \nu_4 \varepsilon$$

$$\sum n = (n_0 + n'_0)\, (\nu_1 + \nu_2 + \varepsilon \Delta\nu) \tag{2}$$

Again,

$$-\frac{\mathrm{d}n_1}{\nu_1} = -\frac{\mathrm{d}n_2}{\nu_2} = \frac{\mathrm{d}n_3}{\nu_3} = \frac{\mathrm{d}n_4}{\nu_4} = (n_0 + n'_0)\,\mathrm{d}\varepsilon$$

$$\mathrm{d}\sum n = \mathrm{d}n_1 + \mathrm{d}n_2 + \mathrm{d}n_3 + \mathrm{d}n_4 = (n_0 + n'_0)\, \Delta\nu\, \mathrm{d}\varepsilon \tag{3}$$

From Eqs. (1), (2) and (3),

$$\frac{\mathrm{d}K}{K} - \Delta\nu \frac{\mathrm{d}p}{p} + \Delta\nu\, (n_0 + n'_0)\, \Delta\nu\, \mathrm{d}\varepsilon$$

$$= \nu_3 \frac{(n_0 + n'_0)\, \nu_3\, \mathrm{d}\varepsilon}{n_3} + \nu_4 \frac{(n_0 + n'_0)\, \nu_4\, \mathrm{d}\varepsilon}{n_4}$$

$$-\nu_1 \frac{(n_0 + n'_0)\, \nu_1\, \mathrm{d}\varepsilon}{n_1} - \nu_2 \frac{-(n_0 + n'_0)\, \nu_2\, \mathrm{d}\varepsilon}{n_2}$$

$$\frac{\mathrm{d}K}{K} - \Delta\nu \frac{\mathrm{d}p}{p} = (n_0 + n'_0)\left[\frac{\nu_3^2}{n_3} + \frac{\nu_4^2}{n_4} + \frac{\nu_1^2}{n_1} + \frac{\nu_2^2}{n_2} - \frac{(\Delta\nu)^2}{\Sigma n}\right]\mathrm{d}\varepsilon$$

$$= \frac{n_0 + n'_0}{\Sigma n_{\mathrm{K}}}\left[\frac{\nu_1^2}{x_1} + \frac{\nu_2^2}{x_2} + \frac{\nu_3^2}{x_3} + \frac{\nu_4^2}{x_4} - (\Delta\nu)^2\right]\mathrm{d}\varepsilon$$

$$\frac{\mathrm{d}(\ln K - \Delta\nu \ln p)}{\mathrm{d}\varepsilon} = \frac{n_0 + n'_0}{\Sigma n_{\mathrm{K}}}\frac{1}{\Psi}$$

where

$$\frac{1}{\Psi} = \frac{\nu_1^2}{x_1} + \frac{\nu_2^2}{x_2} + \frac{\nu_3^2}{x_3} + \frac{\nu_4^2}{x_4} - (\Delta\nu)^2$$

L.H.S.

$$= \frac{\mathrm{d}}{\mathrm{d}\varepsilon} \ln K p^{-\Delta\nu} = \frac{\mathrm{d}}{\mathrm{d}\varepsilon} \ln \frac{x_3^{\nu_3}\, x_4^{\nu_4}}{x_1^{\nu_1}\, x_2^{\nu_2}}\, p^{\Delta\nu} p^{-\Delta\nu}$$

$$\frac{\mathrm{d}}{\mathrm{d}\varepsilon} \ln \frac{x_3^{\nu_3}\, x_4^{\nu_4}}{x_1^{\nu_1}\, x_2^{\nu_2}} = \frac{n_0 + n'_0}{\Sigma n_{\mathrm{K}}}\frac{1}{\Psi} \tag{4}$$

where x's are the equilibrium values.

Now,

$$\frac{1}{\Psi} = \frac{\nu_1^2}{x_1} + \frac{\nu_2^2}{x_2} + \frac{\nu_3^2}{x_3} + \frac{\nu_4^2}{x_4} - (\Delta\nu)^2$$

$$= \left[\frac{\nu_1^2}{n_1} + \frac{\nu_2^2}{n_2} + \frac{\nu_3^2}{n_3} + \frac{\nu_4^2}{n_4}\right] \Sigma n - (\Delta\nu)^2$$

$$= \left[\frac{\nu_1^2}{\nu_1 (1-\varepsilon)} + \frac{\nu_2^2}{\nu_2 (1-\varepsilon)} + \frac{\nu_3^2}{\nu_3 \varepsilon} + \frac{\nu_4^2}{\nu_4 \varepsilon}\right] \frac{\Sigma n}{n_0 + n'_0} - (\Delta\nu)^2$$

$$= \left[\frac{\nu_1\varepsilon + \nu_2\varepsilon + \nu_3 - \nu_3\varepsilon + \nu_4 - \nu_4\varepsilon}{\varepsilon(1-\varepsilon)}\right](\nu_1 + \nu_2 + \varepsilon\Delta\nu) - (\Delta\nu)^2$$

$$= \frac{(\nu_3+\nu_4)-\varepsilon\Delta\nu}{\varepsilon(1-\varepsilon)}\,[(\nu_1 + \nu_2) + \varepsilon\Delta\nu] - (\Delta\nu)^2$$

$$= \frac{(\nu_1 + \nu_2)(\nu_3 + \nu_4) - \varepsilon\Delta\nu(\nu_1 + \nu_2) + (\nu_3+\nu_4) - \varepsilon^2(\Delta\nu)^2}{\varepsilon(1-\varepsilon)} - (\Delta\nu)^2$$

$$= \frac{(\nu_1+\nu_2)(\nu_3+\nu_4)+\varepsilon(\Delta\nu)^2 - \varepsilon^2(\Delta\nu)^2}{\varepsilon(1-\varepsilon)} - (\Delta\nu)^2$$

$$= \frac{(\nu_1+\nu_2)(\nu_3+\nu_4)}{\varepsilon(1-\varepsilon)} + \frac{\varepsilon(1-\varepsilon)(\Delta\nu)^2}{\varepsilon(1-\varepsilon)} - (\Delta\nu)^2 = \frac{(\nu_1+\nu_2)(\nu_3+\nu_4)}{\varepsilon(1-\varepsilon)}$$

$$\therefore \frac{d}{d\varepsilon}\ln\left[\frac{x_3^{\nu_3}\,x_4^{\nu_4}}{x_1^{\nu_1}\,x_2^{\nu_2}}\right]_{\varepsilon=\varepsilon_e} = \frac{(n_0+n'_0)}{\sum n_K}\cdot\frac{(\nu_1+\nu_2)(\nu_3+\nu_4)}{\varepsilon(1-\varepsilon)}$$

Proved.

Example 16.5

For the dissociation of nitrogen tetraoxide according to the equation

$$N_2O_4 \rightleftharpoons 2NO_2$$

Show that the degree of dissociation at equilibrium is

$$e = \frac{V_e}{V_0} - 1$$

where V_0 = initial volume and V_e volume at equilibrium. At 50°C and 0.124 atm, there is a 77.7% increase in volume when equilibrium is reached. Find the value of the equilibrium constant.

Solution $N_2O_4 \rightleftharpoons 2NO_2$

Starting with n_0 moles of N_2O_4 at temperature T and pressure p, the initial volume V_0 is

$$V_0 = n_0\,\frac{\bar{R}T}{p}$$

If V_e denotes the volume at equilibrium, the temperature and pressure remaining the same, then

$$V_e = [n_0(1 - \varepsilon_e) + 2n_0\,\varepsilon_e]\,\frac{\bar{R}T}{p}.$$

where ε_e is the value of the degree of dissociation at equilibrium. This can be written:

$$V_e = (1 + \varepsilon_e)V_0$$

or, $$\varepsilon_e = \frac{V_e}{V_0} - 1$$ Proved.

Given $V_e/V_0 = 1.777,\ \varepsilon_e = 0.777.$

Now, $n_1 = n_0\,\nu_1(1 - \varepsilon_e) = n_0(1 - \varepsilon_e)$

$$n_3 = n_0\,\nu_3, \varepsilon_e = n_0 \cdot 2\varepsilon_e$$

$$\Sigma n = n_0(1 + \varepsilon_e)$$

$$x_1 = \frac{1-\varepsilon_e}{1+\varepsilon_e}, x_3 = \frac{2\varepsilon_e}{1+\varepsilon_e}$$

$$K = \left[\frac{x_3^{\nu_3} \cdot x_4^{\nu_4}}{x_1^{\nu_1} \cdot x_2^{\nu_2}}\right]_{\varepsilon=\varepsilon_e} p^{\nu_3+\nu_4-\nu_1-\nu_2}$$

$$= \frac{\left[\dfrac{2\varepsilon_e}{1+\varepsilon_e}\right]^2}{\left[\dfrac{1-\varepsilon_e}{1+\varepsilon_e}\right]} p^{2-1} = \frac{4\varepsilon_e^2}{1-\varepsilon_e^2} p$$

$$= \frac{4\times(0.777)^2}{1-(0.777)^2}\ 0.124 = \frac{0.2994}{0.3963} = 0.755 \text{ atm}$$ *Ans.*

Example 16.6

Consider the equilibrium mixture of H_2O vapour, H_2 and O_2 caused by the dissociation of 1 g mol of H_2O at 1 atm and 1900 K. If $\Delta H = 250, 560$ J/g mol, $\varepsilon_e = 3.2 \times 10^{-3}$, estimate $C_P - \sum n_k C_{P_k}$.

Solution $H_2O \rightleftharpoons H_2 + \frac{1}{2}O_2$

$$\nu_1 = 1, \nu_2 = 0, \nu_3 = 1, \nu_4 = \frac{1}{2}$$

$$n_1 = n_0\nu_1(1 - \varepsilon_e) = n_0(1 - \varepsilon_e),$$

$$n_3 = n_0\,\nu_3\,\varepsilon_e = n_0\,\varepsilon_e, n_4 = n_0\,\nu_4\,\varepsilon_e = n_0\,\frac{\varepsilon_e}{2}$$

$$\sum n = n_0\left(1+\frac{\varepsilon_e}{2}\right)$$

Now, $$\frac{d}{d\varepsilon_e}\ln\frac{x_3^{\nu_3}\,x_4^{\nu_4}}{x_1^{\nu_1}\,x_2^{\nu_2}} = \frac{n_0}{\Sigma n}\frac{(\nu_1+\nu_2)(\nu_3+\nu_4)}{\varepsilon_e(1-\varepsilon_e)}$$

$$C_P - \sum n_k c_{P_k} = \frac{(\Delta H)^2(1+\varepsilon_e/2)\varepsilon_e(1+\varepsilon_e)}{\bar{R}T^2(\nu_1+\nu_2)(\nu_3+\nu_4)}$$

$$= \frac{(250{,}560)^2 \times 3.2\times10^{-3}}{8.3143\times(1900)^2\times\dfrac{3}{2}} = 4.462 \text{ J/g mol K}$$ *Ans.*

Example 16.7

The products of combustion of an unknown hydrocarbon C_xH_y have the following composition as measured by an Orsat apparatus:

CO_2 8.0%, CO 0.9%, O_2 8.8% and N_2 82.3%

Determine: (a) the composition of the fuel, (b) the air-fuel ratio, and (c) the percentage excess air used.

Solution Let 'a' moles of oxygen be supplied per mole of fuel. The chemical reaction can be written as follows.

$$C_xH_y + aO_2 + 3.76\,aN_2 \rightarrow 8CO_2 + 0.9CO + 8.8O_2 + 82.3N_2 + \frac{y}{2}\,H_2O$$

Oxygen balance gives

$$2a = 16 + 0.9 + 17.6 + \frac{y}{2} \qquad (1)$$

Nitrogen balance gives

$$3.76a = 82.3$$

$$\therefore \quad a = 21.89$$

By substituting the value of a in Eq. (1),

$$y = 18.5$$

Carbon balance gives

$$x = 8 + 0.9 = 8.9$$

Therefore, the chemical formula of the fuel is $C_{8.9}H_{18.5}$.

(a) The composition of the fuel is

$$\%\text{ carbon} = \frac{8.9 \times 12}{8.9 \times 12 + 18.5 \times 1} \times 100 = 58.23\%$$

$$\therefore \quad \%\text{ hydrogen} = 14.77\%$$

(b) Air fuel ratio $= \dfrac{32a + 3.76a \times 28}{12x + y}$

$$= \frac{21.89(32 + 3.76 \times 28)}{12\times 8.9 + 18.5} = \frac{3005}{125.3} = 24$$

(c) % Excess air used

$$= \frac{8.8 \times 32}{21.89 \times 32 - 8.8\times 32} \times 100 = 67.22\%$$

Example 16.8

Determine the heat transfer per kg mol of fuel for the following reaction

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O\ (l)$$

The reactants and products are each at a total pressure of 100 *kPa and* 25°*C.*

Solution By the first law

$$Q_{\text{C.V.}} + \sum_R n_i\bar{h}_i = \sum_P n_e\bar{h}_e$$

From Table C in the appendix

$$\sum_R n_i\bar{h}_i = (\bar{h}_f)_{CH_4} = -74{,}874\text{ kJ}$$

$$\sum_P n_e\bar{h}_e = (\bar{h}_f^0)_{CO_2} + 2(\bar{h}_f^0)_{H_2O(l)}$$

$$= -393{,}522 + 2(-285{,}838) = -965{,}198\text{ kJ} \qquad \textit{Ans.}$$

$$\therefore \quad Q_{\text{C.V.}} = -965{,}198 - (-74{,}873) = -890{,}325\text{ kJ} \qquad \textit{Ans.}$$

Example 16.9

A gasoline engine delivers 150 kW. The fuel used is C_8H_{18} (1) and it enters the engine at 25°C. 150% theoretical air is used and it enters at 45°C. The products of combustion leave the engine at 750 K, and the heat transfer from the engine is 205 kW. Determine the fuel consumption per hour, if complete combustion is achieved.

Solution The stoichiometric equation (Fig. Ex. 16.9) gives

$$C_8H_{18}\,(l) + 12.5\,O_2 + 3.76 \times 12.5N_2 \rightarrow 8CO_2 + 9H_2O + 3.76 \times 12.5N_2$$

With 150% theoretical air $C_8H_{18} + 18.7O_2 + 70N_2 \rightarrow 8CO_2 + 9H_2O + 6.25O_2 + 70N_2$

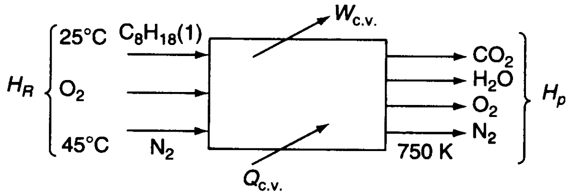


Fig. Ex. 16.9

$$H_R = \sum_R n_i[\bar{h}_f^0 + (\bar{h}_T^0 - \bar{h}_{298})]_i$$

$$= 1\,(\bar{h}_f)_{C_8H_{18\,(l)}} + 18.7[\bar{h}_f^0 + (\bar{h}_T - \bar{h}_{298})]_{O_2} + 70\ [\bar{h}_f^0 + (\bar{h}_T - \bar{h}_{298})]_{N_2}$$

$$= (-\ 249{,}952) + 18.7 \times 560 + 70 \times 540 = -\ 201{,}652 \text{ kJ/kg mol fuel}$$

$$H_P = 8[-\ 393{,}522 + 20288] + 9\,[-\ 241{,}827 + 16087] + 6.25\,[14171] + 70\,[13491]$$

$$= 8\,(-\ 373{,}234) + 9\,[-\ 225{,}740] + 6.25\,(14171) + 70\,[13491]$$

$$= -\ 3{,}884{,}622 \text{ kJ/kg mol fuel}$$

Energy output from the engine, $W_{C.V.} = 150$ kW

$$Q_{C.V.} = -\ 205 \text{ kW}$$

Let n kg mol of fuel be consumed per second.

By the first law

$$n(H_R - H_P) = W_{C.V.} - Q_{C.V.}$$

$$n\,[-\ 201{,}652 + 3{,}884{,}662] = 150 - (-\ 205) = 355 \text{ kW}$$

$$\therefore \qquad n = \frac{355 \times 3600}{3683010} \text{ kg mol/h} = 0.346 \text{ kg mol/h}$$

$\therefore$ Fuel consumption rate = 0.346 × 114 = 39.4 kg/h *Ans.*

Example 16.10

Determine the adiabatic flame temperature when liquid octane at 25°C is burned with 300% theoretical air at 25°C in a steady flow process.

Solution The combustion equation with 300% theoretical air is given by

$$C_8H_{18}(l) + 3(12.5)O_2 + 3(12.5)\,(3.76)\,N_2 \rightarrow$$
$$\rightarrow 8CO_2 + 9H_2O + 25O_2 + 141N_2$$

By the first law

$$H_R = H_P$$

$$\sum_R n_i[\bar{h}_f^0 + \Delta\bar{h}]_i = \sum_P n_e[\bar{h}_f^0 + \Delta\bar{h}]_e$$

Now $H_R = (\bar{h}_{f^0})_{C_8H_{18}(l)} = -\ 249{,}952$ kJ/kg mol fuel

$$H_P = \sum_P n_e[\bar{h}_{f^0} + \Delta\bar{h}]_e = 8\,[-\ 393{,}522 + \Delta\,\bar{h}_{CO_2}]$$

$$+\ 9[-\ 241{,}827 + \Delta\bar{h}_{H_2O}] + 25\ \Delta\bar{h}_{O_2} + 141\ \Delta\bar{h}_{N_2}$$

The exit temperature, which is the adiabatic flame temperature, is to be computed by trial and error satisfying the above equation.

Let $T_e = 1000$ K, then

$$H_P = 8(-\ 393{,}522 + 33{,}405) + 9(-\ 241{,}827 + 25{,}978)$$

$$+\ 25(22.707) + 141(21{,}460)$$

$$= -\ 1226{,}577 \text{ kJ/kg mol}$$

If $T_e = 1200$ K

$$H_P = 8(-\ 393{,}522 + 44{,}484) + 9(-241{,}827 + 34{,}476)$$

$$+\ 25(29{,}765) + 141(28{,}108)$$

$$= +\ 46{,}537 \text{ kJ/kg mol}$$

If $T_e = 1100$ K,

$$H_P = 8(-\ 393{,}522 + 38{,}894) + 9(-\ 241{,}827 + 30{,}167)$$

$$+\ 25(26{,}217) + 141(24{,}757)$$

$$= -\ 595{,}964 \text{ kJ/kg mol}$$

Therefore, by interpolation, the adiabatic flame temperature is 1182 K *Ans.*

Example 16.11

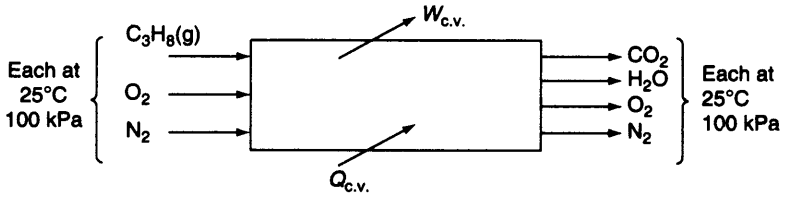
(a) Propane (g) at 25°C and 100 kPa is burned with 400% theoretical air at 25°C and 100 kPa. Assume that the reaction occurs reversibly at 25°C, that the oxygen and nitrogen are separated before the reaction takes place (each at 100 kPa, 25°C), that the constituents in the products are separated, and that each is at 25°C, 100 kPa. Determine the reversible work for this process. (b) If the above reaction occurs adiabatically, and each constituent in the products is at 100 kPa pressure and at the adiabatic flame temperature, compute (a) the increase is entropy during combustion, (b) the irreversibility of the process, and (c) the availability of the products of combustion.

Solution The combustion equation (Fig. Ex. 16.11) is

$$C_3H_8(g) + 5(4)O_2 + 5(4)\,(3.76)N_2 \rightarrow$$

$$3CO_2 + 4H_2O(g) + 15O_2 + 75.2N_2$$

(a) $W_{rev} = \sum_R n_i\bar{g}_i - \sum_P n_e\bar{g}_e$



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From Table 16.6

$$W_{rev} = (\bar{g}_{f^0})_{C_3H_8(g)} - 3(\bar{g}_{f^0})_{CO_2} - 4(\bar{g}_{f^0})_{H_2O(g)}$$

$$= -23{,}316 - 3(-394{,}374) - 4(-228{,}583)$$

$$= 2{,}074{,}128 \text{ kJ/kg mol} \qquad \textit{Ans.}$$

$$= \frac{2{,}074{,}128}{44.097} = 47{,}035.6 \text{ kJ/kg} \qquad \textit{Ans.}$$

(b) $H_R = H_P$

$$(\bar{h}_{f^0})_{C_3H_8(g)} = 3(\bar{h}_{f^0} + \Delta\bar{h})_{CO_2} + 4(\bar{h}_{f^0} + \Delta\bar{h})_{H_2O(g)} + 15\,\Delta\bar{h}_{O_2} + 75.2\,\Delta\bar{h}_{N_2}$$

From Table 16.1

$$-103{,}847 = 3(-393{,}522 + \Delta\bar{h})_{CO_2} + 4(-241{,}827 + \Delta\bar{h})_{H_2O(g)} + 15\,\Delta\bar{h}_{O_2} + 75.2\,\Delta\bar{h}_{N_2}$$

Using Table C in the appendix, and by trial and error, the adiabatic flame temperature is found to be 980 K.

The entropy of the reactants

$$S_R = \sum_R \left(n_i \bar{s}_i^0\right)_{298} = \left(\bar{s}^0_{C_3H_8(g)} + 20\bar{s}^0_{O_2} + 75.2\bar{s}^0_{N_2}\right)_{298}$$

$$= 270.019 + 20\,(205.142) + 75.2\,(191.611)$$

$$= 18{,}782.01 \text{ kJ/kg mol K}$$

The entropy of the products

$$S_P = \sum_P \left(n_e \bar{s}_e^0\right)_{980}$$

$$= \left(3\bar{s}^0_{CO_2} + 4\bar{s}^0_{H_2O(g)} + 15\bar{s}^0_{O_2} + 75.2\bar{s}^0_{N_2}\right)_{980}$$

$$= 3(268.194) + 4(231.849) + 15(242.855) + 75.2(227.485)$$

$$= 22{,}481.68 \text{ kJ/kg mol K}$$

∴ The increase in entropy during combustion

$$S_P - S_R = 3{,}699.67 \text{ kJ/kgmol K} \qquad \textit{Ans.}$$

The irreversibility of the process

$$I = T_0\left[\sum_P n_e \bar{s}_e - \sum_R n_i \bar{s}_i\right]$$

$$= 298 \times 3699.67 = 1{,}102{,}501.66 \text{ kJ/kgmol}$$

$$= \frac{1{,}102{,}501.66}{44.097} = 25{,}001 \text{ kJ/kg}$$

The availability of combustion products

$$\psi = W_{rev} - I$$

$$= 47{,}035.6 - 25{,}001$$

$$= 22{,}034.6 \text{ kJ/kg}$$ *Ans.*

Example 16.12

Determine the chemical exergy of (a) carbon, (b) hydrogen, (c) methane, (d) carbon monoxide, (e) liquid methanol, (f) nitrogen, (g) oxygen, (h) carbon dioxide and (i) water in kJ/kg, in respect of the environment in which the gas phase obeys the ideal gas model.

Environment:

$T_0 = 298.15$ *K*, $p_0 = 1$ *atm*, N_2 75.67%, O_2 20.35%, $H_2O(g)$ 3.12%, CO_2 0.03% *and others* 0.83%, *by volume.*

Solution (a) Carbon: $C + O_2 \rightarrow CO_2$

$$\bar{a}_{ch} = \left[\bar{g}_c + \bar{g}_{O_2} - \bar{g}_{CO_2}\right]_{T_0,\, P_0} + \bar{R}T_0 \ln \frac{x_{O_2}}{x_{CO_2}}$$

$$= 0 + 0 - (-394{,}380) + 8.3143 \times 298.15 \ln \frac{0.2035}{0.0003}$$

$$= 410{,}541 \text{ kJ/k mol} = 34{,}212 \text{ kJ/kg}$$

(b) Hydrogen: $H_2 + \frac{1}{2} O_2 \rightarrow H_2O$

$$\bar{a}_{ch} = \left[\bar{g}_{H_2} + \frac{1}{2}\bar{g}_{O_2} - \bar{g}_{H_2O}\right]_{P_0,\, T_0} + \bar{R}T_0 \ln \frac{(x_{O_2})^{1/2}}{x_{H_2O(g)}}$$

$$= 0 + 0 - (-228590) + 8.3143 \times 298.15 \ln \frac{(0.2035)^{1/2}}{0.0312}$$

$$= 235{,}212 \text{ kJ/kg mol} = 116{,}557 \text{ kJ/kg.}$$ *Ans.*

(c) Methane: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

$$\bar{a}_{ch} = \left[\bar{g}_{CH_4} + 2\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O(g)}\right]$$

$$+ \bar{R}T_0 \ln \frac{x_{O_2}^2}{(x_{CO_2})^1 (x_{H_2O})^2}$$

$$= -50{,}790 + 0 - (-394{,}380) - 2(-228{,}590)$$

$$+ 8.3143 \times 298.15 \ln \frac{(0.2035)^2}{(0.0003)(0.0312)^2}$$

$$= 830{,}174 \text{ kJ/kg mol} = 51{,}756 \text{ kJ/kg}$$ *Ans.*

(d) Carbon monoxide: $CO + \frac{1}{2} O_2 \rightarrow CO_2$

$$\bar{a}_{ch} = \left[\bar{g}_{CO} + \frac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2}\right] - \bar{R}T_0 \ln \frac{x_{O_2}^{1/2}}{x_{CO_2}}$$

$$= -\,137{,}150 + 0 - (-\,394{,}380) + 8.3143 \times 298.15 \ln \frac{(0.2035)^{1/2}}{0.0003}$$

$$= 275{,}364 \text{ kJ/kg mol} = 9831 \text{ kJ/kg}$$ *Ans.*

(e) Liquid methanol: $CH_3OH(l) + 1.5O_2 \rightarrow CO_2 + 2H_2O$

$$\bar{a}_{ch} = \left[\bar{g}_{CH_3OH(l)} + 1.5\,\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\,\bar{g}_{H_2O(g)}\right] + \bar{R}T_0 \ln \frac{x_{O_2}^{1.5}}{\left(x_{CO_2}\right)\left(x_{H_2O}\right)^2}$$

$$= -\,166{,}240 + 0 - (-\,394{,}380) - 2(-\,228{,}590)$$

$$+\, 8.3143 \times 298.15 \ln \frac{(0.2035)^{1.5}}{(0.0003)(0.0312)^2}$$

$$= 716{,}647 \text{ kJ/kg mol} = 22{,}360 \text{ kJ/kg}$$ *Ans.*

(f) Nitrogen: $N_2 \rightarrow N_2$

$$\bar{a}_{ch} = \bar{R}T_0 \ln \frac{1}{x_{N_2}} = 8.3143 \times 298.15 \ln \frac{1}{0.7567}$$

$$= 691.1 \text{ kJ/k mol} = 24.7 \text{ kJ/kg}$$ *Ans.*

(g) Oxygen: $O_2 \rightarrow O_2$

$$\bar{a}_{ch} = \bar{R}T_0 \ln \frac{1}{x_{O_2}} = 8.3143 \times 298.15 \ln \frac{1}{0.2035}$$

$$= 3947 \text{ kJ/k mol} = 123.3 \text{ kJ/kg}$$ *Ans.*

(h) Carbon dioxide:

$$\bar{a}_{ch} = \bar{R}T_0 \ln \frac{1}{x_{CO_2}} = 8.3143 \times 298.15 \ln \frac{1}{0.0003}$$

$$= 20{,}108 \text{ kJ/kg mol} = 456.9 \text{ kJ/kg}$$ *Ans.*

(i) Water: $H_2O(l) \rightarrow H_2O(g)$

$$\bar{a}_{ch} = \bar{g}_{H_2O(l)} - \bar{g}_{H_2O(g)} + \bar{R}T_0 \ln \frac{1}{x_{H_2O}}$$

$$= -\,237{,}180 - (-\,228{,}590) + 8.3143 \times 298.15 \ln \frac{1}{0.0312}$$

$$= 4.9 \text{ kJ/kg mol} = 0.272 \text{ kJ/kg}$$ *Ans.*

Example 16.13

Liquid octane (C_8H_{18}) at 25°C, 1 atm and a mass flow rate of 0.57 kg/h enters a small internal combustion engine operating at steady state. The fuel burns with air entering the engine in a separate stream at 25°C, 1 atm. Combustion products exit at 670 K, 1 atm with a dry molar analysis of 11.4% CO_2, 2.9% CO, 1.6% O_2 and 84.1% N_2. If the engine develops power at the rate of 1 kW, determine (a) the rate of heat transfer from the engine in kW, (b) the second law efficiency of the engine. Use the environment as given below and neglect KE and PE effects.

Environment:

1 *atm*, 298.15 *K and* N_2 75.67%, O_2 20.25%, H_2O(g) 3.12%, CO_2 0.03% *and other* 0.83%, *by volume.*

Solution

$$C_8H_{18}(l) + a(O_2 + 3.76N_2) \rightarrow b\,[0.114CO_2 + 0.029\,CO + 0.016O_2 + 0.841N_2] + CH_2O$$

By carbon balance,

$$b(0.114 = 0.029) = 8$$

$$b = 55.9$$

By hydrogen balance, $18 = 2C \quad C = 9$

By oxygen balance, $a = b \times 0.114 + \frac{b}{2} \times 0.029 + b \times 0.016 + \frac{C}{2}$

$$= b\,(0.114 + 0.0145 + 0.016) + \frac{9}{2} = 12.58$$

The combustion equation becomes,

$$C_8H_{18}(l) + 12.58(O_2 + 3.76N_2) \rightarrow 55.9[0.114\,CO_2 + 0.029\,CO + 0.016\,O_2 + 0.841N_2] + 9H_2O$$

By first law, $Q + H_R = W + H_P$

$$\frac{Q_{CV}}{\dot{n}_{fuel}} = \frac{\dot{W}_{CV}}{\dot{n}_{fuel}} - (-249{,}910) + 55.9\,[0.114(-393{,}520 + 25284) + 0.029$$

$$(-110{,}530 + 11{,}089) + 0.016(1151.5) + 0.841\,(11016) + 9(-241{,}820 + 13064)$$

$$= \frac{\dot{W}_{CV}}{\dot{n}_{fuel}} - 3{,}845{,}872 \text{ kJ/kg mol } C_8H_{18}(l)$$

$$\dot{Q}_{CV} = \dot{W}_{CV} - \dot{n}_{fuel} \times 3{,}845{,}872 \text{ kW}$$

$$= 1 - 3{,}845{,}872\,\frac{\text{kJ}}{\text{kg mol}} \times \frac{0.57 \text{ kg/h}}{3600 \text{ s/h}} \times \frac{1 \text{ kg mol}}{114.22 \text{ kg}}$$

$$= 1 - 5.33 = -4.33 \text{ kW}$$

Ans. (a)

(b) Chemical exergy of C_8H_{18} was found to be 5,407,843 kJ/kg mol (Art. 16.19).

$$\eta_{II} = \frac{\dot{W}_{CV}}{\dot{n}_{fuel} \times \bar{a}_{ch}}$$

$$= \frac{1 \text{ kW}}{5407843\,\frac{\text{kJ}}{\text{kg mol}} \times \frac{0.57 \text{ kg}}{3600 \text{ s}} \times \frac{1 \text{ kg mol}}{114.22 \text{ kg}}}$$

$$= \frac{1}{7.496} = 0.133 \text{ or } 13.3\%$$

Ans. (b)

Review Questions

16.1 What are stoichiometric coefficients?

16.2 What is the degree of reaction? What are its limiting values?

16.3 When does a chemical reaction reach equilibrium?

16.4 What do you understand by the equilibrium constant of a chemical reaction?

16.5 What is the law of mass action?

16.6 What is the heat of reaction? When is it positive and when negative?

16.7 What is the van't Hoff equation?

16.8 Give Nernst's equation.

16.9 What is thermal ionization?

16.10 What is the significance of Saha's equation?

16.11 What do you understand by the standard Gibbs function change?

16.12 What is chemical affinity?

16.13 Define fugacity and activity.

16.14 How can the heat capacity of reacting gases in equilibrium be estimated?

16.15 What is stoichiometric air?

16.16 What do you understand by the enthalpy of formation?

16.17 Define adiabatic flame temperature. How is it estimated?

16.18 What is enthalpy of combustion? What is internal energy of combustion?

16.19 What do you understand by higher heating value and lower heating value of a fuel?

16.20 State Planck's formulation of the third law of thermodynamics.

16.21 What is the Gibbs function of formation?

Problems

16.1 Starting with n_0 moles of NO, which dissociates according to the equation

$$NO \rightleftharpoons \frac{1}{2}N_2 + \frac{1}{2}O_2$$

show that at equilibrium

$$K = \frac{1}{2} \cdot \frac{\varepsilon_e}{1-\varepsilon_e}$$

16.2 A mixture of $n_0\, \nu_1$ moles of A_1 and $n_0\, \nu_2$ moles of A_2 at temprature T and pressure p occupies a volume V_0. When the reaction

$$\nu_1 A_1 + \nu_2 A_2 \rightleftharpoons \nu_3 A_3 + \nu_4 A_4$$

has come to equilibrium at the same T and p, the volume is V_e. Show that

$$\varepsilon_e = \frac{V_e - V_0}{V_0} \cdot \frac{\nu_1 + \nu_2}{\nu_3 + \nu_4 - \nu_1 - \nu_2}$$

16.3 The equilibrium constant of the reaction

$SO_3 \rightleftharpoons SO_2 + \frac{1}{2}O_2$ has the following values

T	800 K	900 K	1000 K	1105 K
K	0.0319	0.153	0.540	1.59

Determine the average heat of dissociation graphically. *Ans.* 94 MJ/k mol

16.4 At high temperature the potassium atom is ionized according to the equation $K \rightleftharpoons K^+ + e^-$. The values of the equilibrium constant at 3000 K and 3500 K are 8.33×10^{-6} and 1.33×10^{-4} respectively. Compute the average heat of reaction in the given temperature range. *Ans.* 483,819 J/g mol

16.5 (a) Show that $\Delta G = \Delta H + T\left(\frac{\partial \Delta G}{\partial T}\right)_p$

(b) Show that $\Delta G = -\bar{R}T \ln \frac{x_3^{\nu_3} \cdot x_4^{\nu_4}}{x_1^{\nu_1} \cdot x_2^{\nu_2}}$

where the x's are equilibrium values.

16.6 When 1 kg mol of HI dissociates according to the reaction

$$HI \rightleftharpoons \frac{1}{2}H_2 + \frac{1}{2}I_2$$

at $T = 675$ K, $K = 0.0174$, and $\Delta H = 5910$ kJ/kg mol.

Calculate $(\partial \varepsilon_e / \partial T)_p$ at this temperature.

Ans. 0.505×10^{-4} K^{-1}

16.7 Calculate the degree of ionization of cesium vapour at 10^{-6} atm at the two temperatures of 2260 and 2520 K. *Ans.* 0.393, 0.808

16.8 In the case of ionization of a monatomic gas, show that:

(a) $$\frac{G - G(\text{mia})}{\bar{R}T} = \varepsilon\left[\ln\frac{\varepsilon^2}{1-\varepsilon^2} - \ln\frac{\varepsilon_e^2}{1-\varepsilon_e^2}\right] + \ln\frac{1-\varepsilon}{1+\varepsilon} - \ln\frac{1-\varepsilon_e}{1+\varepsilon_e}$$

(b) At $\varepsilon = 0$, $\dfrac{G_0 - G\,(\text{main})}{\bar{R}T} = -\ln\dfrac{1-\varepsilon_e}{1+\varepsilon_e}$

(c) At $\varepsilon = 1$, $\dfrac{G_1 - G\,(\text{main})}{\bar{R}T} = \ln\dfrac{1}{4} - \ln\dfrac{\varepsilon_e^2}{(1+\varepsilon_e)^2}$

(d) Plot $\left[G - G(\text{min})\right]/(230\bar{R}T)$ against ε for the ionization of cesium vapour at 2260 K and 10^{-6} atm.

16.9 Prove that, for a mixture of reacting ideal gases in equilibrium

(a) $$\left(\frac{\partial V}{\partial T}\right)_T = -\frac{V}{T} - \frac{(n_0 + n_0')\bar{R}T(\Delta\nu)^2}{p^2 \cdot \dfrac{d}{d\varepsilon_e}\ln\dfrac{x_3^{\nu_3}\cdot x_4^{\nu_4}}{x_1^{\nu_1}\cdot x_2^{\nu_2}}}$$

(b) $$\left(\frac{\partial V}{\partial T}\right)_p = \frac{V}{T} + \frac{(n_0 + n_0')\Delta\nu\Delta H}{pT\dfrac{d}{d\varepsilon_e}\ln\dfrac{x_3^{\nu_3}\cdot x_4^{\nu_4}}{x_1^{\nu_1}\cdot x_2^{\nu_2}}}$$

(c) $$\left(\frac{\partial p}{\partial T}\right)_{\varepsilon_e} = \frac{p\Delta H}{\bar{R}T^2\Delta\nu}$$

16.10 Prove that, for a mixture of reacting ideal gases in equilibrium,

$$dS = \sum n_k\left[\sum x_k c_{pk} + \frac{\psi(\Delta H)^2}{\bar{R}T^2}\right]\frac{dT}{T} - \bar{R}\sum n_k\left[1 + \frac{\psi\Delta H\Delta\gamma}{\bar{R}T^2}\right]\frac{dp}{p}$$

16.11 Oxygen dissociates according to the relation $O_2 \rightarrow 2O$. Show that the equilibrium constant is given by

$$K = \frac{4\varepsilon_e^2 p}{1-\varepsilon_e^2}$$

where ε_e is the degree of dissociation. Obtain an expression for the rate of variation of ε_e with temperature, $\left(\dfrac{\partial\varepsilon_e}{\partial T}\right)_p$, in terms of ε_e, $\dfrac{dK}{dT}$ and the total pressure p. At a temperature of 3800 K, $K = 1$ atm and $\dfrac{\Delta H}{\bar{R}} = 59$ K, calculate the value of ε and $\left(\dfrac{\partial\varepsilon_e}{\partial T}\right)_p$ at a pressure of 1 atm.

16.12 (a) Starting with n_0 moles of water vapour which dissociates according to the equation $H_2O \rightleftharpoons H_2 + \frac{1}{2}O_2$, show that at equilibrium

$$K = \frac{\varepsilon_e^{3/2}}{(2+\varepsilon_e)^{1/2}(1-\varepsilon_e)}\cdot p^{1/2}$$

(b) At an average temperature of 1900 K, the slope of the graph of log K against $1/T$ for the dissociation of water vapour is found to be −13,000. Determine the heat of dissociation. Is it exothermic or endothermic?

Ans. 248,885 kJ/kg mol, endothermic

16.13 The volumetric composition of the 'dry' products of combustion of an unknown hydrocarbon fuel, C_xH_y, gives: CO_2 12.1%, O_2 3.8%, CO 0.9% and N_2 83.2%. Determine (a) the chemical formula of the fuel, (b) the air fuel ratio, and (c) the percentage of excess air used.

Ans. (a) $C_{13}H_{23}$, (b) 17.03, (c) 18%

16.14 Liquid octane is burned with air in a combustion test, and the dry volumetric analysis of the products reveals the following composition: CO_2 11%, O_2 3.6%, CO 1.5% and N_2 83.9%. Compute the actual air-fuel ratio used during the test and the percentage excess air. *Ans.* 17.267, 14.19%

16.15 Propane (C_3H_8) is reacted with air in such a ratio that an analysis of the products of combustion gives CO_2 11.5%, O_2 2.7% and CO 0.7%. What is the percent theoretical air used during the test?

Ans. 111%

16.16 Carbon monoxide and 300% theoretical air enter a steady flow combustor at 400 K and a low pressure. The energy released by the reaction heats the product gases to 1400 K. If the combustion is complete, estimate the heat gained or lost through the walls of the combustor.

Ans. −17,360 kJ/kg mol

16.17 Liquid octane enters the combustion chamber of a gas turbine at 25°C and air enters from the compressor at 227°C. It is determined that 98% of the carbon in the fuel burns to form CO_2 and the remaining 2% burns to form CO. The temperature of the products is limited to 827°C. Estimate the air-fuel ratio used and the percentage excess air.
Ans. 66,338%

16.18 A mixture of methane and oxygen, in the proper ratio for complete combustion and at 25°C and 1 atm, reacts in a constant volume calorimeter bomb. Heat is transferred until the products of combustion are at 400 K. Determine the heat transfer per mole of methane.
Ans. −794414 kJ/kg mol

16.19 Liquid hydrazine (N_2H_4) and oxygen gas, both at 25°C, 0.1 MPa are fed to a rocket combustion chamber in the ratio of 0.5 kg O_2/kg N_2H_4. The heat transfer from the chamber to the surroundings is estimated to be 100 kJ/kg N_2H_4. Determine the temperature of the products, assuming only H_2O, H_2, and N_2 to be present. The enthalpy of the formation of N_2H_4 (l) is + 50,417 kJ/kg mol.
Ans. 2855 K

If saturated liquid oxygen at 90 K is used instead of 25°C oxygen gas in the combustion process, what will the temperature of the products be?

16.20 Liquid ethanol (C_2H_5OH) is burned with 150% theoretical oxygen in a steady flow process. The reactants enter the combustion chamber at 25°C, and the products are cooled and leave at 65°C, 0.1 MPa. Calculate the heat transfer per kg mol of ethanol. The enthalpy of formation of C_2H_5OH (l) is −277,634 kJ/kg mol.

16.21 A small gas turbine uses C_8H_{18}(l) for fuel and 400% theoretical air. The air and fuel enter at 25°C and the combustion products leave at 900 K. If the speicific fuel consumption is 0.25 kg/s per MW output, determine the heat transfer from the engine per kg mol of fuel, assuming complete combustion. *Ans.* −48.830 kJ/kg mol.

16.22 Hydrogen peroxide (H_2O_2) enters a gas generator at the rate of 0.1 kg/s, and is decomposed to steam and oxygen. The resulting mixture is expanded through a turbine to atmospheric pressure, as shown in Fig. P. 16.22. Determine the power output of the turbine and heat transfer rate in the gas generator. The enthalpy of formation of H_2O_2(l) is −187,583 kJ/kg mol. *Ans.* 38.66 kW, −83.3 kW

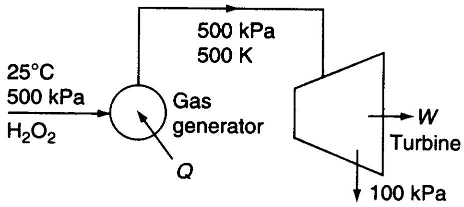


Fig. P. 16.22

16.23 An internal combustion engine burns liquid octane and uses 150% theoretical air. The air and fuel enter at 25°C, and the products leave the engine exhaust ports at 900 K. In the engine 80% of the carbon burns to CO_2 and the remainder burns to CO. The heat transfer from this engine is just equal to the work done by the engine. Determine (a) the power output of the engine if the engine burns 0.006 kg/s of fuel, and (b) the composition and the dew point of the product of combustion.

16.24 Gaseous butane at 25°C is mixed with air at 400 K and burned with 400% theoretical air. Determine the adiabatic flame temperature.

16.25 A mixture of butane and 150% theoretical air enters a combustion chamber of 25°C, 150 kPa, and the products of combustion leave at 1000 K, 150 kPa. Determine the heat transfer from the combustion chamber and the irreversibility for the process.

16.26 The following data are taken from the test of a gas turbine:

Fuel—C_4H_{10}(g) 25°C, 0.1 MPa
Air—300% theoretical air at 25°C, 0.1 MPa
Velocity of inlet air—70 m/s
Velocity of products at exit—700 m/s
Temperature and pressure of products—900 K, 0.1 MPa

Assuming complete combustion, determine (a) the net heat transfer per kg mol of fuel, (b) the net increase of entropy per kg mol of fuel, and (c) the irreversibility for the process.

16.27 Calculate the equilibrium composition if argon gas is heated in an arc to 10,000 K, 1 kPa, assuming the plasma to consist of A, A^+, e^-. The equilibrium constant for the reaction

$$A \rightleftharpoons A^+ + e^-$$

at this temperature is 0.00042.

16.28 Methane is to be burned with oxygen in an adiabatic steady-flow process. Both enter the combustion chambar at 298 K, and the products leave at 70 kPa. What percent of excess O_2 should be used if the flame temperature is to be 2800 K? Assume that some of the CO_2 formed dissociates to CO and O_2, such that the products leaving the chamber consist of CO_2, CO, H_2O and O_2 at equilibrium.

16.29 (a) Octane burns with the stoichiometric amount of air. Determine the air fuel ratio and the partial pressure of CO_2 in the dry products of combustion when the total pressure is 1 atm. (b) If 25% excess air is used in burning octane and combustion is complete determine the dry volumetric analysis of the products of combustion and the dew point temperature of the products.

Ans. (a) 15.11, 0.1455 atm, (b) CO_2 11.45%, O_2 4.47%, N_2 84.0 8; 47.356°C

16.30 Methane gas at 25°C, 1 atm and a volumetric flow rate of 27 m^3/h enters a furnace operating at steady state. The methane burns completely with 140% theoretical air, entering at 127°C at 1 atm. Products of combustion leave at 427°C, 1 atm. Determine (a) the volumetric flow rate of air in m^3/h, (b) the rate of heat transfer from the furnace in kW. Neglect KE and PE changes.

Ans. (a) 483.03 m^3/h, (b) −202.5 kW

16.31 Octane gas at 25°C, 1 atm enters a jet engine and burns completely with 300% theoretical air at 25°C, 1 atm. Products of combustion leave at 990 K, 1 atm. If the fuel and air enter with relatively low air velocities, determine the velocity of the combustion products at exit. Neglect PE effects and heat transfer between the engine and surroundings.

Ans. 641.84 m/s

16.32 Liquid octane (C_8H_{18}) at 25°C, 1 atm enters the combustior of a simple open gas turbine power plant and burns completely with 400% theoretical air entering the compressor at 25°C, 1 atm. Products of combustion exit the turbine at 627°C, 1 atm. If the rate of heat transfer from the gas turbine is estimated as 15% of the net power developed, determine the net power developed in kJ per kg mol of fuel. Kinetic and potential energy effects are negligible.

Ans. 439,750 kJ/kg mol

16.33 Gaseous ethanol (C_6H_5OH) at 25°C, 1 atm enters a reactor operating at steady state and burns completely with 130% theoretical air entering at 25°C, 1 atm. Products of combustion exit at 127°C, 1 atm. If the rate of heat transfer from the reactor is 900 kW, determine the mass flow rate of the fuel, in kg/s, neglecting the KE and PE effects.

Ans. 0.034 2 kg/s

16.34 Carbon monoxide at 25°C, 1 atm enters an insulated reactor operating at steady state and reacts completely with the theoretical amount of air entering in a separate steam at 25°C, 1 atm. The products exit as mixture at 1 atm. Determine in kJ/kg mol of carbon (a) the availability entering with CO, (b) the availability exiting with the products, (c) the irreversibility rate and (d) the second law efficiency. Assume the environment as given in Example 16.13.

Ans. (a) 275,364 kJ/k mol CO, (b) 226,105 kJ/k mol CO, (c) 49,259 kJ/mol CO, (d) 82.1%

16.35 Liquid octane (C_8H_{18}) at 25°C, 1 atm enters an adiabatic reactor operated at steady state and burns completely with air entering at 227°C, 1 atm. If the combustion products exit at 1127°C, determine the percent excess air used, neglecting KE and PE effects.

Ans. 168.5%

CHAPTER 17

Compressible Fluid Flow

A fluid is defined as a substance which continuously deforms under the action of shearing forces. Liquids and gases are termed as fluids. A fluid is said to be *incompressible* if its density (or specific volume) does not change (or changes very little) with a change in pressure (or temperature or velocity). Liquids are incompressible. A fluid is said to be *compressible* if its density changes with a change in pressure or temperature or velocity. Gases are compressible. The effect of compressibility must be considered in flow problems of gases. Thermodynamics is an essential tool in studying compressible flows, because of which Theodore von Karman suggested the name 'Aerothermodynamics' for the subject which studies the dynamics of compressible fluids.

The basic principles in compressible flow are:

(a) Conservation of mass (continuity equation)
(b) Newton's second law of motion (momentum principle)
(c) Conservation of energy (first law of thermodynamics)
(d) Second law of thermodynamics (entropy principle)
(e) Equation of state.

For the first two principles, the student is advised to consult a book on fluid mechanics, and the last three principles have been discussed in the earlier chapters of this book.

17.1 VELOCITY OF PRESSURE PULSE IN A FLUID

Let us consider an infinitesimal pressure wave initiated by a slight movement of a piston to the right (Fig. 17.1) in a pipe of uniform cross-section. The pressure wave front propagates steadily with a velocity c, which is known as the velocity of sound, sonic velocity or acoustic velocity. The fluid near the piston will have a slightly increased pressure and will be slightly more dense, than the fluid away from the piston.

To simplify the analysis, let the observer be assumed to travel with the wave front to the right with the velocity c. Fluid flows steadily from right to left and as it passes through the wave front, the velocity is reduced from c to c-d$\bar{V}$. At the same time, the pressure rises from p to p + dp and the density from ρ to ρ + dρ.

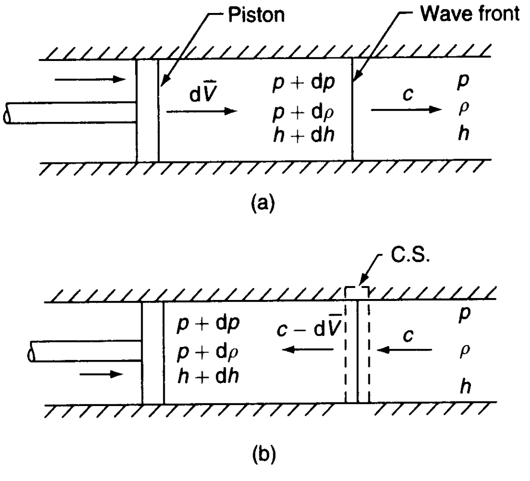


Fig. 17.1 *Diagram illustrating sonic velocity* (a) *Stationary observer* (b) *Observer travelling with the wave front*

The continuity equation for the control volume gives

$$\rho A c = (\rho + \mathrm{d}\rho) A (c - \mathrm{dV})$$

$$\rho c = \rho c - \rho\, \mathrm{dV} + c\mathrm{d}\rho - \mathrm{d}\rho \cdot \mathrm{dV}$$

Neglecting the product $\mathrm{d}\rho \cdot \mathrm{dV}$, both being very small

$$\rho\, \mathrm{dV} = c\mathrm{d}\rho \tag{17.1}$$

The momentum equation for the control volume gives

$$[p - (p + \mathrm{d}p)] A = w\,[(c - \mathrm{dV}) - c]$$

$$-\mathrm{d}p\, A = \rho A c\,(c - \mathrm{dV} - c)$$

$$\mathrm{d}p = \rho\, c\mathrm{dV} \tag{17.2}$$

From Eqs (17.1) and (17.2) $$\frac{\mathrm{d}p}{c} = c\mathrm{d}\rho$$

$$\therefore \quad c = \sqrt{\frac{\mathrm{d}p}{\mathrm{d}\rho}}$$

Since the variations in pressure and temperature are negligibly small and the change of state is so fast as to be essentially adiabatic, and in the absence of any internal friction or viscosity, the process is reversible and isentropic. Hence, the sonic velocity is given by

$$c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \tag{17.3}$$

No fluid is truly incompressible, although liquids show little change in density. The velocity of sound in common liquids is of the order of 1650 m/s.

17.1.1 Velocity of Sound in an Ideal Gas

For an ideal gas, in an isentropic process

$$pv^{\gamma} = \text{constant} \quad \text{or} \quad \frac{p}{\rho^{\gamma}} = \text{constant}$$

By logarithmic differentiation (i.e., first taking logarithm and then differentiating)

$$\frac{\mathrm{d}p}{p} - \gamma \frac{\mathrm{d}\rho}{\rho} = 0$$

$$\therefore \quad \frac{\mathrm{d}p}{\mathrm{d}\rho} = \gamma \frac{p}{\rho}$$

Since $$c^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho} \quad \text{and} \quad p = \rho RT$$

$$c^2 = \gamma RT$$

or $$c = \sqrt{\gamma RT} \tag{17.4}$$

where $$R = \text{characteristic gas constant} = \frac{\text{Universal gas constant}}{\text{Molecular weight}}$$

The lower the molecular weight of the fluid and higher the value of γ, the higher is the sonic velocity at the same temperature. c is a thermodynamic property of the fluid.

17.1.2 Mach Number

The Mach number, M, is defined as the ratio of the actual velocity V to the sonic velocity c.

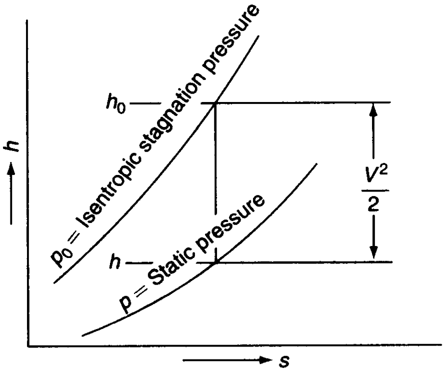
$$\mathrm{M} = \frac{\mathrm{V}}{c}$$

When $M > 1$, the flow is supersonic, when $M < 1$, the flow is subsonic, and when $M = 1$, the flow is sonic.

17.2 STAGNATION PROPERTIES

The isentropic *stagnation state* is defined as the state a fluid in motion would reach if it were brought to rest isentropically in a steady-flow, adiabatic, zero work output device. This is a reference state in a compressible fluid flow and is commonly designated with the subscript zero. The stagnation enthalpy h_0 (Fig. 17.2) is related to the enthalpy and velocity of the moving fluid by

$$h_0 = h + \frac{V^2}{2} \qquad (17.5)$$



Stagnation state

For an ideal gas, $h = h(T)$ and c_p is constant. Therefore

$$h_0 - h = c_p (T_0 - T) \qquad (17.6)$$

From Eqs (17.5) and (17.6)

$$c_p(T_0 - T) = \frac{V^2}{2}$$

$$\frac{T_0}{T} = 1 + \frac{V^2}{2c_pT}$$

The properties without any subscript denote static properties.

Since $$c_p = \frac{\gamma R}{\gamma - 1}$$

$$\frac{T_0}{T} = 1 + \frac{V^2(\gamma - 1)}{2\gamma RT}$$

Using Eq. (17.4) and the Mach number

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \qquad (17.7)$$

The stagnation pressure p_0 is related to the Mach number and static pressure in the case of an ideal gas by the following equation

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)} \qquad (17.8)$$

Similarly, $$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)} \qquad (17.8a)$$

17.3 ONE-DIMENSIONAL STEADY ISENTROPIC FLOW

From a one dimensional point of view, the three most common factors which tend to produce continuous changes in the state of a flowing stream are

(a) Changes in cross-sectional area
(b) Wall friction
(c) Energy effects, such as external heat exchange, combustion, etc.

A study will first be made of the effects of area change in the absence of friction and energy effects. The process, which has been called isentropic flow, might aptly be termed as simple area change.

A nozzle is any duct which increases the kinetic energy of a fluid at the expense of its pressure. A diffuser is a passage through which a fluid loses kinetic energy and gains pressure. The same duct or passage may be either a nozzle or diffuser depending upon the end conditions across it. A nozzle or diffuser with both a converging and a diverging section is shown in Fig. 17.3. The minimum section is known as the throat. For the control volume shown in Fig. 17.3, since stagnation enthalpy and stagnation temperature do not change in adiabatic flow

$$h_0 = h + \frac{V^2}{2}$$

$$dh = -V\,dV \tag{17.9}$$

From the property relation $Tds = dh - v\,dp$

For isentropic flow

$$dh = \frac{dp}{\rho} \tag{17.10}$$

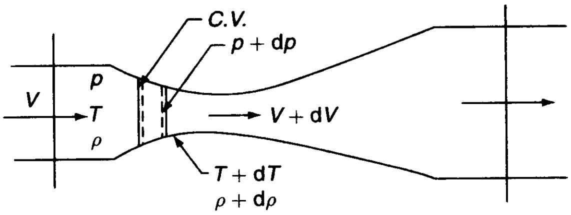


Fig. 17.3 *Reversible adiabatic flow through a nozzle*

From Eqs (17.9) and (17.10)

$$dp = -\rho V\,dV \tag{17.11}$$

or

$$\frac{dp}{dV} < 0 \tag{17.12}$$

As pressure decreases, velocity increases, and vice versa.

The continuity equation gives

$$w = \rho A V$$

By logarithmic differentiation

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

$$\therefore \quad \frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho}$$

Substituting from Eq. (17.11)

$$\frac{dA}{A} = \frac{dp}{\rho V^2} - \frac{d\rho}{\rho} = \frac{dp}{\rho V^2}\left[1 - V^2 \frac{d\rho}{dp}\right]$$

or

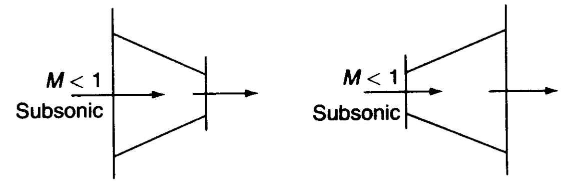
$$\frac{dA}{A} = \frac{dp}{\rho V^2}(1 - M^2) \tag{17.13}$$

Also

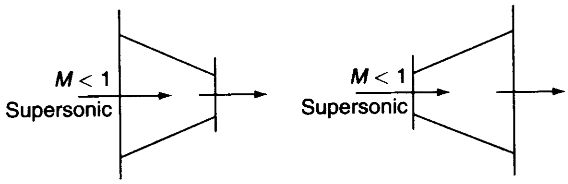
$$\frac{dA}{A} = (M^2 - 1)\frac{dV}{V} \tag{17.14}$$

When $M < 1$, i.e., the inlet velocity is subsonic, as now area A decreases, the pressure decreases and velocity increases, and when flow area A increases, pressure increases and velocity decreases. So for subsonic flow, a convergent passage becomes a nozzle (Fig. 17.4a) and a divergent passage becomes a diffuser (Fig. 17.4b).

When $M > 1$, i.e., when the inlet velocity is supersonic, as now area A decreases, pressure increases and velocity decreases, and as flow area A increases, pressure decreases and velocity increases. So for supersonic flow, a convergent passage is a diffuser (Fig. 17.4c) and a divergent passage is a nozzle (Fig. 17.4d).



(a) Nozzle (b) Diffuser



(c) Diffuser (d) Nozzle

Fig. 17.4 *Effect of area change in subsonic and supersonic flow at inlet to duct*

17.4 CRITICAL PROPERTIES–CHOKING IN ISENTROPIC FLOW

Let us consider the mass rate of flow of an ideal gas through a nozzle. The flow is isentropic

$$w = \rho A \mathrm{V}$$

or

$$\frac{w}{A} = \frac{p}{RT} \cdot cM = \frac{p}{RT}\sqrt{\gamma RT} \cdot M = \frac{p}{p_0} \cdot p_0 \sqrt{\frac{T_0}{T}} \cdot \sqrt{\frac{1}{T_0}}\sqrt{\frac{\gamma}{R}} \cdot M$$

$$= \left(\frac{T_0}{T}\right)^{-\gamma/\gamma-1}\left(\frac{T_0}{T}\right)^{1/2} \cdot \frac{p_0}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}} \cdot M = \sqrt{\frac{\gamma}{R}}\frac{p_0 M}{\sqrt{T_0}} \cdot \frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)^{(\gamma+1)/2(\gamma-1)}} \qquad (17.15)$$

Since p_0, T_0, γ and R are constant, the discharge per unit area w/A is a function of M only. There is a particular value of M when w/A is a maximum. Differentiating Eq. (17.15) with respect to M and equating it to zero,

$$\frac{\mathrm{d}(w/A)}{\mathrm{d}M} = \sqrt{\frac{\gamma}{R}} \cdot \frac{p_0}{\sqrt{T_0}} \frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)^{(\gamma+1)/2(\gamma-1)}}$$

$$+\sqrt{\frac{\gamma}{R}} \cdot \frac{p_0 M}{\sqrt{T_0}}\left[-\frac{\gamma+1}{2(\gamma-1)}\right]\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}-1}\left(\frac{\gamma-1}{2} \cdot 2M\right) = 0$$

$$1 - \frac{M^2(\gamma+1)}{2\left(1+\frac{\gamma-1}{2}M^2\right)} = 0$$

$$M^2(\gamma+1) = 2 + (\gamma-1)M^2$$

$$M^2 = 1 \quad \text{or} \quad M = 1$$

So, the discharge w/A is maximum when $M = 1$.

Since $\mathrm{V} = cM = \sqrt{\gamma RT} \cdot M$, by logarithmic differentiation

$$\therefore \qquad \frac{\mathrm{dV}}{\mathrm{V}} = \frac{\mathrm{d}M}{M} + \frac{1}{2}\frac{\mathrm{d}T}{T} \qquad (17.16)$$

and

$$\frac{T}{T_0} = \left(1+\frac{\gamma-1}{2}M^2\right)^{-1}$$

by logarithmic differentiation

$$\therefore \qquad \frac{\mathrm{d}T}{T} = -\frac{(\gamma-1)M^2}{1+\frac{\gamma-1}{2}M^2}\frac{\mathrm{d}M}{M} \qquad (17.17)$$

From Eqs (17.16) and (17.17)

$$\therefore \qquad \frac{\mathrm{dV}}{\mathrm{V}} = \frac{1}{1+\frac{\gamma-1}{2}M^2} \cdot \frac{\mathrm{d}M}{M} \qquad (17.18)$$

From Eqs (17.14) and (17.18)

$$\frac{\mathrm{d}A}{A}\left(\frac{1}{M^2-1}\right) = \frac{1}{1+\frac{\gamma-1}{2}M^2}\frac{\mathrm{d}M}{M}$$

$$\therefore \qquad \frac{\mathrm{d}A}{A} = \frac{(M^2-1)\mathrm{d}M}{M\left(1+\frac{\gamma-1}{2}M^2\right)} \qquad (17.19)$$

By substituting $M = 1$ in any one of the Eqs (17.13), (17.14) or (17.19), $dA = 0$ or $A =$ constant. So $M = 1$ *occurs only at the throat and nowhere else, and this happens only when the discharge is the maximum.*

If the convergent-divergent duct acts as a nozzle, in the divergent part also, the pressure will fall continuously to yield a continuous rise in velocity. The velocity of the gas is subsonic before the throat, becomes sonic at the throat, and then becomes supersonic till its exit in isentropic flow, provided the exhaust pressure is low enough. The reverse situation prevails when the inlet velocity is supersonic. The whole duct then becomes a diffuser. The transition from subsonic flow to supersonic flow and vice versa can occur only when the compressible fluid flows through a throat and the exit pressure is maintained at the appropriate value.

When $M = 1$, the discharge is maximum and the nozzle is said to be choked. The properties at the throat are then termed as critical properties and these are designated by a superscript asterisk (*). Substituting $M = 1$ in Eq. (17.7)

$$\frac{T_0}{T^*} = 1 + \frac{\gamma-1}{2}M^2 = 1 + \frac{\gamma-1}{2} = \frac{\gamma+1}{2}$$

$$\therefore \quad \frac{T^*}{T_0} = \frac{2}{\gamma+1} \tag{17.20}$$

The *critical pressure ratio* p^*/p_0 is then given by

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1} \tag{17.20a}$$

For diatomic gases, like air, $\gamma = 1.4$

$$\frac{p^*}{p_0} = \left(\frac{2}{2.4}\right)^{1.4/0.4} = 0.528$$

The critical pressure ratio for air is 0.528. For superheated steam, $\gamma = 1.3$ and p^*/p_0 is 0.546.

For air, $$\frac{T^*}{T_0} = 0.833$$

and $$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} = 0.634$$

By substituting $M = 1$ in Eq. (17.15)

$$\frac{w}{A^*} = \sqrt{\frac{\gamma}{R}} \cdot \frac{p_0}{\sqrt{T_0}} \cdot \frac{1}{\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/2(\gamma-1)}} \tag{17.21}$$

Dividing Eq. (17.21) by Eq. (17.15)

$$\frac{A}{A^*} = \left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/2(\gamma-1)} \frac{1}{M} \tag{17.22}$$

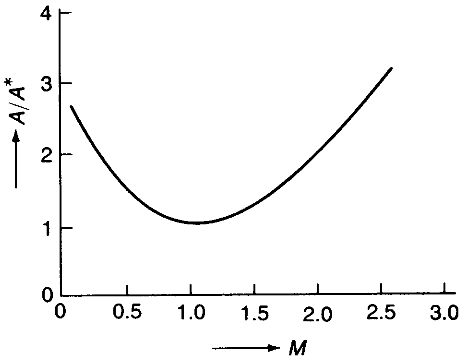


Fig. 17.5 *Area ratio as a function of mach number in an isentropic nozzle*

The area ratio A/A^* is the ratio of the area at the point where the Mach number is M to the throat area A^*. Figure 17.5 shows a plot of A/A^* vs. M, which shows that a subsonic nozzle is converging and a supersonic nozzle is diverging.

17.4.1 Dimensionless Velocity, M*

Since the Mach number M is not proportional to the velocity alone and it tends towards infinity at high speeds, one more dimensionless parameter M^* is often used, which is defined as

$$M^* = \frac{V}{c^*} = \frac{V}{V^*} \tag{17.23}$$

where $c^* = \sqrt{\gamma R T^*} = V^*$

$\therefore$

$$M^{*2} = \frac{V^2}{c^{*2}} = \frac{V^2}{c^2} \cdot \frac{c^2}{c^{*2}}$$

$$= M^2 \cdot \frac{c^2}{c^{*2}}$$

For the adiabatic flow of an ideal gas

$$\frac{V^2}{2} + c_p T = \text{constant} = h_0 = c_p T_0$$

$$\frac{V^2}{2} + \frac{\gamma R T}{\gamma - 1} = \frac{\gamma R T_0}{\gamma - 1}$$

$$\frac{V^2}{2} + \frac{c^2}{\gamma - 1} = \frac{c_0^2}{\gamma - 1}$$

Since

$$\frac{c_0}{c^*} = \sqrt{\frac{\gamma R T_0}{\gamma R T^*}} = \sqrt{\frac{T_0}{T^*}} = \sqrt{\left(\frac{\gamma+1}{2}\right)}$$

$\therefore$

$$\frac{V^2}{2} + \frac{c^2}{\gamma - 1} = \frac{\gamma+1}{2} c^{*2} \frac{1}{\gamma - 1}$$

$$\frac{V^2}{c^{*2}} + \frac{2}{\gamma - 1} \frac{c^2}{c^{*2}} = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2} + \frac{2}{\gamma - 1} \frac{M^{*2}}{M^2} = \frac{\gamma+1}{\gamma-1}$$

On simplification

$$M^{*2} = \frac{\frac{\gamma+1}{2} M^2}{1 + \frac{\gamma - 1}{2} M^2} \tag{17.24}$$

When $M < 1, \quad M^* < 1$

When $M > 1, \quad M^* > 1$

When $M = 1, \quad M^* = 1$

When $M = 0, \quad M^* = 0$

When $M = \infty, \quad M^* = \sqrt{\frac{\gamma+1}{\gamma-1}}$

17.4.2 Pressure Distribution and Choking in a Nozzle

Let us first consider a convergent nozzle as shown in Fig. 17.6, which also shows the pressure ratio p/p_0 along the length of the nozzle. The inlet condition of the fluid is the stagnation state at p_0, T_0, which is assumed to be constant. The pressure at the exit plane of the nozzle is denoted by p_E and the back pressure is p_B which can be varied by the valve. As the back pressure p_B is decreased, the mass flow rate w and the exit plane pressure p_E/p_0 vary, as shown in Fig. 17.7.

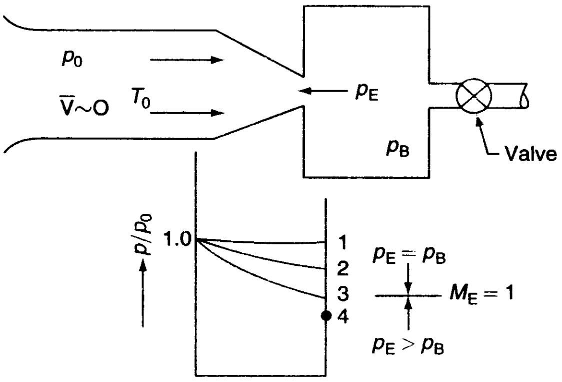


Fig. 17.6 *Compressible flow through a converging nozzle*

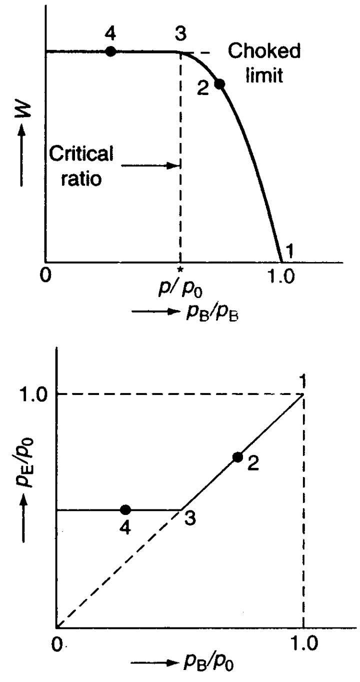


Fig. 17.7 *Mass flow rate and exit pressure as a function of back pressure in a converging nozzle*

When $p_B/p_0 = 1$, there is no flow, and $p_E/p_0 = 1$, as designated by point 1. If the back pressure p_B is now decreased to a value as designated by point 2, such that p_B/p_0 is greater than the critical pressure ratio, the mass flow rate has a certain value, and $p_E = p_B$. The exit Mach number M_E is less than 1. Next the back pressure p_B is lowered to the critical pressure, denoted by point 3. The exit Mach number M_E is now unity, and $p_E = p_B$. When p_B is increased below the critical pressure, indicated by point 4, there is no increase in the mass flow rate, and p_E remains constant at a value equal to critical pressure, and $M_E = 1$. The drop in pressure from p_E to p_B occurs outside the nozzle exit. This is choking limit which means that for given stagnation conditions the nozzle is passing the maximum possible mass flow.

Let us next consider a convergent-divergent nozzle, as shown in Fig. 17.8. Point 1 designates the condition when $p_B = p_0$, and there is no flow. When p_B is lowered to the pressure denoted by point 2, so that p_B/p_0 is less than 1 but greater than the critical pressure ratio, the velocity increases in the convergent section, but $M < 1$ at the throat. The divergent section acts as a subsonic diffuser in which the pressure increases and velocity decreases. Point 3 indicates the back pressure at which $M = 1$ at the throat, but the diverging section acts as a subsonic diffuser in which the pressure increases and velocity decreases. Point 4 indicates one other back pressure for which the flow is isentropic throughout and the diverging section acts as a supersonic nozzle with a continuous decrease in pressure and a continuous increase in velocity, and $p_{E4} = p_{B4}$. This condition of supersonic flow past the throat with the isentropic

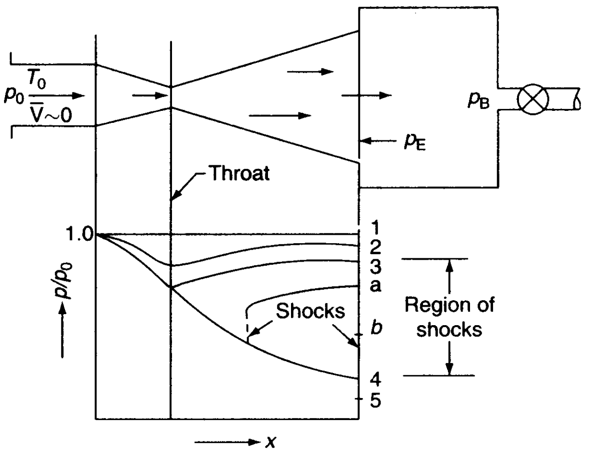


Fig. 17.8 *Compressible flow through a convergent divergent nozzle*

conditions is called the *design pressure ratio of the nozzle.* If the pressure is lowered to 5, no further decrease in exit pressure occurs and the drop in pressure from p_E to p_B occurs outside the nozzle.

Between the back pressures designated by points 3 and 4, flow is not isentropic in the diverging part, and it is accompanied by a highly irreversible phenomenon, known as *shocks.* Shocks occur only when the flow is supersonic, and after the shock the flow becomes subsonic, when the rest of the diverging portion acts as a diffuser. Properties very discontinuously across the shock. When the back pressure is as indicated by point *b* (Fig. 17.8), the flow throughout the nozzle is isentropic, with pressure continuously decreasing and velocity increasing, but a shock appears just at the exit of the nozzle. When the back pressure is increased from *b* to *a*, the shock moves upstream, as indicated. When the back pressure is further increased, the shock moves further upstream and disappears at the nozzle throat where the back pressure corresponds to 3. Since flow throughout is subsonic, no shock is possible.

17.4.3 Gas Tables for Isentropic Flow

The values of M^*, A/A^*, p/p_0, ρ/ρ_0, and T/T_0 computed for an ideal gas having $\gamma = 1.4$ for various values of Mach number M from the Eqs (17.24), (17.22), (17.8), and (17.7) respectively are given in Table D.1 in the appendix. These may be used with advantage for computations of problems of isentropic flow.

17.5 NORMAL SHOCKS

Shock waves are highly localized irreversibilities in the flow. Within the distance of a mean free path of a molecule, the flow passes from a supersonic to a subsonic state, the velocity decreases abruptly, and the pressure rises sharply. Figure 17.9 shows a control surface that includes such a normal shock. Normal shocks may be treated as shock waves perpendicular to the flow. The fluid is assumed to be in thermodynamic equilibrium upstream and downstream of the shock, the properties of which are designated by the subscripts *x* and *y* respectively. For the control surface,

Continuity equation $$\frac{w}{A} = \rho_x V_x = \rho_y V_y = G \qquad (17.25)$$

where G is the mass velocity (kg/m²s).

Momentum equation $$p_x - p_y = \frac{w}{A}(V_y - V_x)$$

$$= \rho_y V^2_y - \rho_x V^2_x$$

$$\therefore \quad p_x + \rho_x V^2_x = p_y + \rho_y V^2_y \qquad (17.26)$$

or $$F_x = F_y \qquad (17.26a)$$

where $F = pA + \rho A V^2$ is known as the *impulse function.*

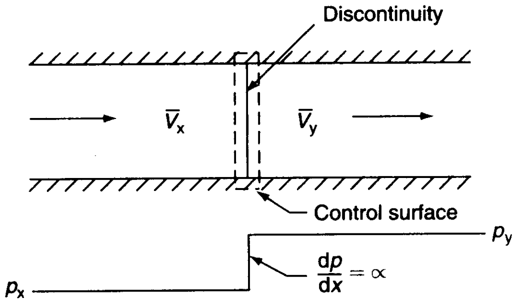


Fig. 17.9 *One dimensional normal shock*

Energy equation $$h_x + \frac{V_x^2}{2} = h_y + \frac{V_y^2}{2} = h_{0x} + h_{0y} = h_0 \qquad (17.27)$$

where h_0 is the stagnation enthalpy on both sides of the shock. Second law states

$$s_y - s_x \geq 0 \qquad (17.28)$$

The equation of state of the fluid may be written implicitly in the form

$$h = h(s, \rho) \qquad (17.29a)$$

$$dh = \left(\frac{\partial h}{\partial s}\right)_\rho ds + \left(\frac{\partial h}{\partial \rho}\right)_s d\rho$$

or $$s = s(p, \rho) \qquad (17.29b)$$

Combining the continuity and energy equations and dropping the subscripts x and y

$$h_0 = h + \frac{G^2}{2\rho^2} = \text{constant} \tag{17.30}$$

or

$$h = h_0 - \frac{G^2}{2\rho^2}$$

Given the values of G and h_0, this equation relates h and ρ (Fig. 17.10). The line representing the locus of points with the same mass velocity and stagnation enthalpy is called a *Fanno line*. The end states of the normal shock must lie on the Fanno line.

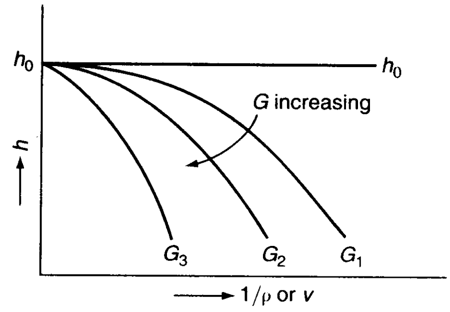


Fig. 17.10 *Fanno line on h-1/ρ coordinate*

The Fanno line may also be represented in the h-s diagram. The upstream properties h_x, ρ_x, p_x and V_x are known. Let a particular value of V_y be chosen, then ρ_y may be computed from the continuity Eq. (17.25), h_y from the energy Eq. (17.27), and s_y from Eq. (17.29a). By repeating the calculation for various values of V_y the Fanno line may easily be constructed (Fig. 17.11). Since the momentum equation has not been introduced, the Fanno line represents states with the same mass velocity and stagnation enthalpy, but not the same value of the impulse function.

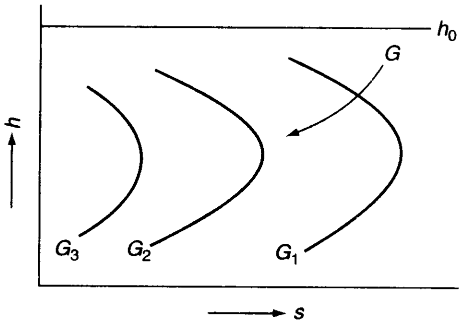


Fig. 17.11 *Fanno line on h-s diagram*

Adiabatic flow in a constant area duct with friction, in a one dimensional model, has both constant G and constant h_0, and hence must follow a Fanno line.

Let us next consider the locus of states which are defined by the continuity Eq. (17.25), the momentum Eq. (17.26) and the equation of state (17.29). The impulse function in this case becomes

$$F = pA + \rho A V^2$$

or the *impulse pressure I* is given by

$$= \frac{F}{A} = p + \rho V^2 = p + \frac{G^2}{\rho} \tag{17.31}$$

Given the values for I and G, the equation relates p and ρ. The line representing the locus of states with the same impulse pressure and mass velocity is called the *Rayleigh line*. The end states of the normal shock must lie on the Rayleigh line, since $I_x = I_y$, and $G_x = G_y$.

The Rayleigh line may also be drawn on the h-s plot. The properties upstream of the shock are all known. The downstream properties are to be known. Let a particular value of V_y be chosen. Then ρ_y may be computed from the continuity Eq. (17.25) and p_y from the momentum Eq. (17.26), and s_y from Eq. (17.29b) may be found. By repeating the calculations for various values of V_y, the locus of possible states reachable from, say, state x and may be plotted, and this is the Rayleigh line (Fig. 17.12).

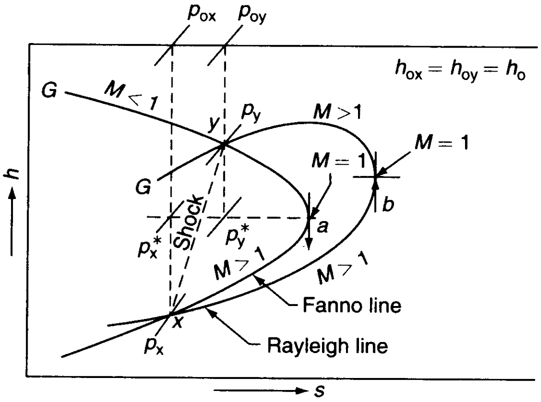


Fig. 17.12 *End states of a normal shock on h-s diagram*

Since the normal shock must satisfy Eqs (17.25), (17.26), (17.27), and (17.29) simultaneously, the end states x and y of the shock must lie at the intersections of the Fanno line and the Rayleigh line for the same G (Fig. 17.12).

The Rayleigh line is also a model for flow in a constant area duct with heat transfer, but without friction.

For an infinitesimal process in the neighbourhood of the point of maximum entropy (point a) on the Fanno line, from the energy equation

$$dh + \mathbf{V}d\mathbf{V} = 0 \tag{17.32}$$

and from the continuity equation

$$\rho\, \mathrm{dV} + \mathrm{Vd}\,\rho = 0 \tag{17.33}$$

From the thermodynamic relation

$$Tds = dh - \mathrm{V}dp$$

or

$$dh = \frac{dp}{\rho} \tag{17.34}$$

By combining Eqs (17.32), (17.33), and (17.34)

$$\frac{dp}{d\rho} + \mathrm{V}\left(-\frac{\mathrm{V}d\rho}{\rho}\right) = 0$$

$$\therefore \qquad \frac{dp}{d\rho} = \mathrm{V}^2$$

or $\mathrm{V} = \sqrt{\left(\dfrac{\partial p}{\partial \rho}\right)_s}$, since the flow is isentropic.

This is the local sound velocity.

So the Mach number is unity at point a. Similarly, it can be shown that at point b on the Rayleigh line, $M = 1$. It may also be shown that the upper branches of the Fanno and Rayleigh lines represent subsonic speeds ($M < 1$) and the lower branches represent supersonic speeds ($M > 1$).

The normal shock always involves a change from supersonic to subsonic speed with a consequent pressure rise, and never the reverse. By the second law, entropy always increases during irreversible adiabatic change.

17.5.1 Normal Shock in an Ideal Gas

The energy equation for an ideal gas across the shock becomes

$$c_p T_x + \frac{\mathrm{V}_x^2}{2} = c_p T_y + \frac{\mathrm{V}_y^2}{2} = c_p T_0$$

Now

$$h_{0x} = h_{0y} = h_0, \text{ and } T_{0x} = T_{0y} = T_0$$

Substituting

$$c_p = \frac{\gamma R}{\gamma - 1},\ c_x = \sqrt{\gamma R T_x}\ , \text{ and } c_y = \sqrt{\gamma R T_y}$$

$$\frac{T_0}{T_x} = 1 + \frac{\gamma - 1}{2} M^2{}_x, \text{ and } \frac{T_0}{T_y} = 1 + \frac{\gamma - 1}{2} M^2{}_y$$

$$\therefore \qquad \frac{T_y}{T_x} = \frac{1 + \dfrac{\gamma - 1}{2} M_x^2}{1 + \dfrac{\gamma - 1}{2} M_y^2} \tag{17.35}$$

Again

$$\rho_x \mathrm{V}_x = \rho_y \mathrm{V}_y$$

$$\therefore \qquad \frac{p_x}{RT_x} \mathrm{V}_x = \frac{p_y}{RT_y} \cdot \mathrm{V}_y$$

or
$$\frac{T_y}{T_x} = \frac{p_y}{p_x}\cdot\frac{V_y}{V_x} = \frac{p_y}{p_x}\cdot\frac{M_y c_y}{M_x c_x} = \frac{p_y}{p_x}\cdot\frac{M_y}{M_x}\sqrt{\frac{T_y}{T_x}}$$

or
$$\frac{T_y}{T_x} = \left(\frac{p_y}{p_x}\right)^2\left(\frac{M_y}{M_x}\right)^2 \tag{17.36}$$

From Eqs (17.35) and (17.36)
$$\frac{p_y}{p_x} = \frac{M_x}{M_y}\sqrt{\frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_y^2}} \tag{17.37}$$

Also
$$p_x + \rho_x V_x^2 = p_y + \rho_y V_y^2$$

or
$$p_x + \frac{\gamma p_x}{\gamma R T_x} V_x^2 = p_y + \frac{\gamma p_y}{R T_y \gamma} V_y^2$$

$$p_x(1 + \gamma M_y^2) = p_y(1 + \gamma M_y^2)$$

∴
$$\frac{p_y}{p_x} = \frac{1+\gamma M_x^2}{1+\gamma M_y^2} \tag{17.38}$$

From Eqs (17.37) and (17.38), upon rearrangement
$$M_y^2 = \frac{M_x^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_x^2 - 1} \tag{17.39}$$

Then from Eqs (17.37), (17.38), and (17.39)
$$\frac{p_y}{p_x} = \frac{2\gamma}{\gamma+1}M_y^2 - \frac{\gamma-1}{\gamma+1} \tag{17.40}$$

and from Eqs (17.35) and (17.39)
$$\frac{T_y}{T_x} = \frac{\left(1+\frac{\gamma-1}{2}M_x^2\right)\left(\frac{2\gamma}{\gamma-1}M_x^2 - 1\right)}{\frac{(\gamma+1)^2}{2(\gamma-1)}M_x^2} \tag{17.41}$$

Then
$$\frac{\rho_y}{\rho_x} = \frac{p_y}{T_y}\cdot\frac{T_x}{p_x} = \frac{\left(\frac{2\gamma}{\gamma+1}M_x^2 - \frac{\gamma-1}{\gamma+1}\right)\left(\frac{(\gamma+1)^2}{2(\gamma-1)}M_x^2\right)}{\left(1+\frac{\gamma-1}{2}M_x^2\right)\left(\frac{2\gamma}{\gamma-1}M_x^2 - 1\right)} \tag{17.42}$$

The ratio of the stagnation pressures is a measure of the irreversibility of the shock process. Now
$$\frac{p_{oy}}{p_{ox}} = \frac{p_{oy}}{p_y}\cdot\frac{p_y}{p_x}\cdot\frac{p_x}{p_{ox}}$$

and
$$\frac{p_{oy}}{p_y} = \left(1+\frac{\gamma-1}{2}M_y^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{p_{\text{ox}}}{p_{\text{x}}} = \left(1+\frac{\gamma-1}{2}M_{\text{x}}^2\right)^{\gamma/(\gamma-1)}$$

$$\therefore \qquad \frac{p_{\text{oy}}}{p_{\text{ox}}} = \left[\frac{\dfrac{\gamma+1}{2}M_{\text{x}}^2}{1+\dfrac{\gamma-1}{2}M_{\text{x}}^2}\right]^{\gamma(\gamma-1)} \Bigg/ \left[\frac{2\gamma}{\gamma+1}M_{\text{x}}^2-\frac{\gamma-1}{\gamma+1}\right]^{1/(\gamma-1)} \tag{17.43}$$

$$\frac{p_{\text{oy}}}{p_{\text{x}}} = \frac{p_{\text{oy}}}{p_{\text{y}}}\cdot\frac{p_{\text{y}}}{p_{\text{x}}} = \left(1+\frac{\gamma-1}{2}M_{\text{y}}^2\right)^{\gamma/(\gamma-1)}\left[\frac{2\gamma}{\gamma+1}M_{\text{x}}^2-\frac{\gamma-1}{\gamma+1}\right] \tag{17.44}$$

For different values of M_{x}, and for $\gamma = 1.4$, the values of $M_{\text{y}}, p_{\text{y}}/p_{\text{x}}, T_{\text{y}}/T_{\text{x}}, \rho_{\text{y}}/\rho_{\text{x}}, p_{\text{oy}}/p_{\text{ox}}$, and $p_{\text{oy}}/p_{\text{x}}$ computed from Eqs (17.39), (17.40), (17.42), (17.43), and (17.44) respectively, are given in Table D.2 in the appendix.

To evaluate the entropy change across the shock, for an ideal gas

$$ds = c_{\text{p}}\frac{dT}{T} - R\frac{dp}{p}$$

$$\therefore \qquad s_{\text{y}} - s_{\text{x}} = c_{\text{p}}\ln\frac{T_{\text{y}}}{T_{\text{x}}} - R\ln\frac{p_{\text{y}}}{p_{\text{x}}} = c_{\text{p}}\left[\ln\frac{T_{\text{y}}}{T_{\text{x}}} - \ln\left(\frac{p_{\text{y}}}{p_{\text{x}}}\right)^{(\gamma-1)/\gamma}\right]$$

since

$$R = \frac{c_{\text{p}}(\gamma-1)}{\gamma}$$

$$\therefore \qquad s_{\text{y}} - s_{\text{x}} = c_{\text{p}}\ln\frac{T_{\text{y}}/T_{\text{x}}}{(p_{\text{y}}/p_{\text{x}})^{(\gamma-1)/\gamma}} = c_{\text{p}}\ln\frac{T_{\text{oy}}/T_{\text{ox}}}{(p_{\text{oy}}/p_{\text{ox}})^{(\gamma-1)/\gamma}} = -R\ln\frac{p_{\text{oy}}}{p_{\text{ox}}} \tag{17.45}$$

The strength of a shock wave, P, is defined as the ratio of the pressure increase to the initial pressure, i.e.

$$P = \frac{p_{\text{y}} - p_{\text{x}}}{p_{\text{x}}} = \frac{p_{\text{y}}}{p_{\text{x}}} - 1$$

Substituting from Eq. (17.40)

$$P = \frac{2\gamma}{\gamma+1}M_{\text{y}}^2 - \frac{\gamma-1}{\gamma+1} - 1$$

$$= \frac{2\gamma}{\gamma+1}(M_{\text{y}}^2 - 1) \tag{17.46}$$

17.6 ADIABATIC FLOW WITH FRICTION AND DIABATIC FLOW WITHOUT FRICTION

It was stated that the Fanno line representing the states of constant mass velocity and constant stagnation enthalpy also holds for adiabatic flow in a constant area duct with friction. For adiabatic flow the entropy must increase in the flow direction. Hence a Fanno process must follow its Fanno line to the right, as shown in Fig. 17.13. Since friction will tend to move the state of the fluid to the right on the Fanno line, the Mach number of subsonic flows increases in the downstream section (Fig. 17.13), and in supersonic flows friction acts to decrease the Mach number. Hence, friction tends to drive the flow to the sonic point.

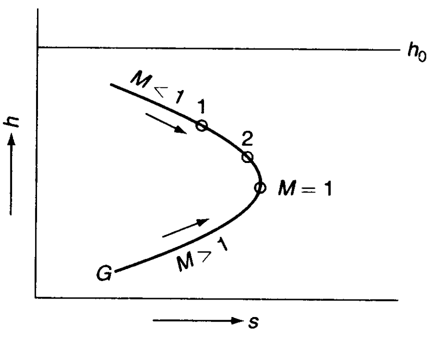


Fig. 17.13 *A fanno line on h-s plot*

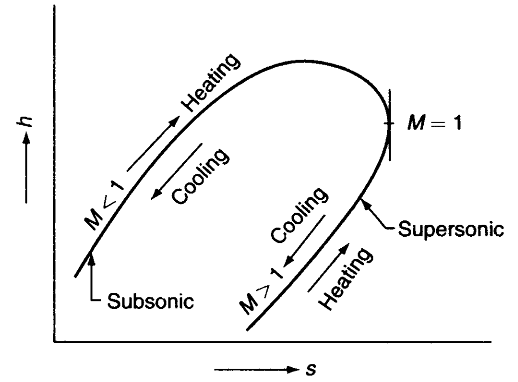


Fig. 17.14 A rayleigh line on h-s plot

Let us consider a short duct with a given h_0 and G, i.e. a given Fanno line, with a given subsonic exit Mach number represented by point 1 in Fig. 17.13. If some more length is added to the duct, the new exit Mach number will be increased due to friction, as represented by, say, point 2. The length of the duct may be further increased till the exit Mach number is unity. Any further increase in duct length is not possible without incurring a reduction in the mass flow rate. Hence subsonic flows can be choked by friction. There is a maximum flow rate that can be passed by a pipe with given stagnation conditions. Choking also occurs in supersonic flow with friction, usually in a very short length. It is thus difficult to use such flows in applications.

Diabatic flows, i.e., flows with heating or cooling, in a constant area duct, in the absence of friction, can be treated by the Rayleigh process (Fig. 17.14). The process is reversible, and the direction of entropy change is determined by the sign of the heat transfer. Heating a compressible flow has the same effect as friction. and the Mach number goes towards unity. Therefore, there is a maximum heat input for a given flow rate which can be passed by the duct, which is then choked. Although the cooling of the fluid increases the flow stagnation pressure with a decrease in entropy, a nonmechanical pump is not feasible by cooling a compressible flow, because of the predominating effect of friction.

Solved Examples

Example 17.1

A stream of air flows in a duct of 100 *mm diameter at a rate of* 1 *kg/s. The stagnation temperature is* 37°*C. At one section of the duct the static pressure is* 40 *kPa. Calculate the Mach number, Velocity, and stagnation pressure at this section.*

Solution

$$T_0 = 37 + 273 = 310\ \text{K}, p = 40\ \text{kPa}, \gamma = 1.4$$

The mass flow rate per unit area is

$$\frac{w}{A} = \rho V = \frac{p}{RT}\sqrt{\gamma RT}\cdot M = \sqrt{\frac{\gamma}{R}}\cdot\frac{pM}{\sqrt{T_0}}\sqrt{\frac{T_0}{T}} = \sqrt{\frac{\gamma}{R}}\cdot\frac{pM}{\sqrt{T_0}}\left(1+\frac{\gamma-1}{2}M^2\right)^{1/2}$$

$$\frac{1}{\frac{\pi}{4}(0.1)^2}\ \text{kg/m}^2\text{s} = \sqrt{\frac{1.4}{0.287\ \text{kJ/kg K}}}\ \frac{40\ \text{kN/m}^2\times M}{\sqrt{310\ \text{K}}}\ (1 + 0.2\,M^2)^{1/2}$$

$$127.39 = \sqrt{\frac{1.4}{0.287\times 10^3\times 310}}\ .\ 40\times 10^3\,M\,(1 + 0.2\,M^2)^{1/2}$$

$$\therefore \quad M(1 + 0.2\,M^2)^{1/2} = 0.803$$

$$M^2\,(1 + 0.2\,M^2) = 0.645$$

$$M^4 + 5M^2 - 3.225 = 0$$

$$M^2 = \frac{-5\pm\sqrt{25+1.29}}{2} = \frac{-5+6.16}{2} = 0.58$$

$$\therefore \quad M = 0.76 \qquad \textit{Ans.}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + 0.2 \times (0.76)^2 = 1.116$$

$$\therefore \quad T = \frac{310}{1.116} = 277.78 \text{ K}$$

$$c = \sqrt{\gamma R T} = \sqrt{1.4 \times 0.287 \times 277.78 \times 10^3} = 334.08 \text{ m/s} \qquad \textit{Ans.}$$

$$\therefore \quad \text{V} = c\,M = 334.08 \times 0.76 = 253.9 \text{ m/s} \qquad \textit{Ans.}$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)} = (1.116)^{1.4/0.4} = 1.468$$

$$\therefore \quad p_0 = 40 \times 1.468 = 58.72 \text{ kPa} \qquad \textit{Ans.}$$

Example 17.2

A conical air diffuser has an intake area of 0.11 *m*^{2} *and an exit area of* 0.44 *m*^{2}. *Air enters the diffuser with a static pressure of* 0.18 *MPa, static temperature of* 37°*C, and* V*elocity of* 267 *m/s. Calculate* (*a*) *the mass flow rate of air through the diffuser,* (*b*) *the Mach number, static temperature, and static pressure of air lea*V*ing the diffuser, and* (*c*) *the net thrust acting upon the diffuser, assuming that its outer surfaces are wetted by atmospheric pressure at* 0.1 *MPa.*

Solution The conical diffuser is shown in Fig. Ex. 17.2. The mass flow rate of air through it

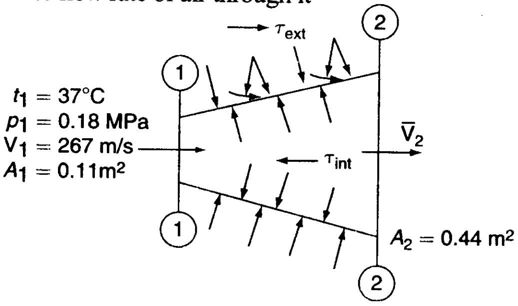


Fig. Ex. 17.2

$$\omega = \rho A \text{V}$$

$$= \frac{p_1}{RT_1} \cdot A_1 \text{V}_1 = \frac{0.18 \times 10^3}{0.287 \times 310}$$

$$\times\ 0.11 \times 267 = 59.42 \text{ kg/s} \quad \textit{Ans.}\text{ (a)}$$

$$c_1 = \sqrt{\gamma R T_1}$$

$$= 20.045\sqrt{T_1} = 20.045\sqrt{310} = 352 \text{ m/s}$$

$$\therefore \quad M_1 = \frac{\text{V}_1}{c_1} = \frac{267}{352} = 0.76$$

From the gas tables for the isentropic flow of air ($\gamma = 1.4$), given in the appendix, when $M_1 = 0.76$

$$\frac{A_1}{A^*} = 1.0570, \quad \frac{p_1}{p_{01}} = 0.68207, \quad \frac{T_1}{T_{01}} = 0.89644$$

$$\frac{F_1}{F^*} = 1.0284$$

Now

$$\frac{A_2}{A_1} = \frac{0.44 \text{ m}^2}{0.11 \text{ m}^2} = 4 = \frac{A_2/A^*}{A_1/A^*}$$

$$\therefore \quad \frac{A_2}{A^*} = 4 \times 1.0570 = 4.228$$

Again, from the isentropic flow tables, when $\frac{A_2}{A^*} = 4.228$

$$M_2 \simeq 0.135, \quad \frac{p_2}{p_{02}} = 0.987, \quad \frac{T_2}{T_{02}} = 0.996, \quad \frac{F_2}{F^*} = 3.46$$

$$\frac{p_2}{p_1} = \frac{p_2/p_{02}}{p_1/p_{01}} = \frac{0.987}{0.682} = 1.447$$

$$\frac{T_2}{T_1} = \frac{T_2/T_{02}}{T_1/T_{01}} = \frac{0.996}{0.89644} = 1.111$$

$$\frac{F_2}{F_1} = \frac{F_2/F^*}{F_1/F^*} = \frac{3.46}{1.0284} = 3.364$$

$\therefore$ $p_2 = 1.447 \times 0.18 = 0.26$ MPa

$T_2 = 1.111 \times 310 = 344.4$ K $= 71.3$°C *Ans.*

Impulse function at inlet

$$F_1 = p_1 A_1 + \rho_1 A_1 V^2{}_1 = p_1 A_1 \left(1 + \frac{1}{RT_1} V_1^2\right) = p_1 A_1 (1 + \gamma M^2{}_1)$$

$$= 0.18 \times 10^3 \times 0.11\,(1 + 1.4 \times 0.76^2) = 35.82 \text{ kN}$$

Internal thrust τ will be from right to left, as shown in Fig. Ex. 17.2.

$$\tau_{\text{int}} = F_2 - F_1 = 3.364\,F_1 - F_1$$

$$= 2.364 \times 35.82 = 84.68 \text{ kN}$$

External thrust is from left to right

$$\tau_{\text{ext}} = p_0(A_2 - A_1) = 0.1 \times 10^3\,(0.44 - 0.11) = 33 \text{ kN}$$

Net thrust $= \tau_{\text{int}} - \tau_{\text{ext}}$

$= 84.68 - 33 = 51.68$ kN *Ans.*(c)

Example 17.3

A conVergent-diVergent nozzle has a throat area 500 mm² and an exit area of 1000 mm². Air enters the nozzle with a stagnation temperature of 360 K and a stagnation pressure of 1 MPa. Determine the maximum flow rate of air that the nozzle can pass, and the static pressure, static temperature, Mach number, and Velocity at the exit from the nozzle, if (a) the diVergent section acts as a nozzle, and (b) the diVergent section acts as a diffuser.

Solution

$$\frac{A_2}{A^*} = \frac{1000}{500} = 2$$

From the isentropic flow tables, when $A_2/A^* = 2$ there are two values of the Mach number, one for supersonic flow when the divergent section acts as a nozzle, and the other for subsonic flow when the divergent section acts as a diffuser, which are $M_2 = 2.197, 0.308$ (Fig. Ex. 17.3).

(a) When $M_2 = 2.197$, $\dfrac{p_2}{p_0} = 0.0939$, $\dfrac{T_2}{T_0} = 0.5089$

$\therefore$ $p_2 = 0.0939 \times 1000 = 93.9$ kPa

$T_2 = 0.5089 \times 360 = 183.2$ K

$c_2 = \sqrt{\gamma R T_2} = 20.045\sqrt{183.2} = 271.2$ m/s

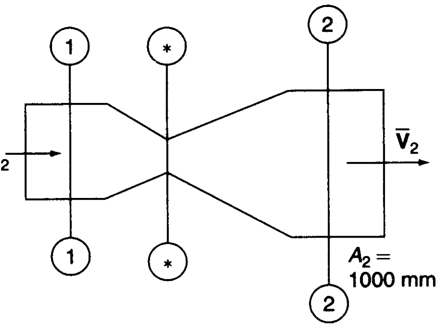
$\therefore$ $V_2 = 271.2 \times 2.197 = 596$ m/s *Ans.*

Mass flow rate $w = A^* \rho^* V^* = \rho_2 A_2 V_2 = \rho_1 A_1 V_1$

For air $\frac{p^*}{p_0} = 0.528$ and $\frac{T^*}{T_0} = 0.833$

$$\rho^* = \frac{p^*}{RT^*} = \frac{0.528 \times 1000}{0.287 \times 0.833 \times 360}$$
$$= 6.13 \text{ kg/m}^3$$

$$T^* = 360 \times 0.833 = 300 \text{ K}$$
$$V^* = \sqrt{\gamma R T^*}$$
$$= 20.045\sqrt{300} = 347.2 \text{ m/s}$$
$$\therefore \quad w = (500 \times 10^{-6}) \times 6.13 \times 347.2$$
$$= 1.065 \text{ kg/s}$$



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Ans.

(b) When $M = 0.308$, $\frac{p_2}{p_0} = 0.936$, $\frac{T_2}{T_0} = 0.9812$

$$p_2 = 0.936 \times 1000 = 936 \text{ kPa}$$
$$T_2 = 0.9812 \times 360 = 353.2 \text{ K}$$
$$c_2 = \sqrt{\gamma R T_2} = 20.045\sqrt{353.2} = 376.8 \text{ m/s}$$
$$V_2 = 376.8 \times 0.308 = 116 \text{ m/s}$$
$$w = 1.065 \text{ kg/s}$$

Ans.

Example 17.4

When a Pitot tube is immersed in a supersonic stream, a curVed shock waVe is formed ahead of the Pitot tube mouth. Since the radius of the curVature of the shock is large, the shock may be assumed to be a normal shock. After the normal shock, the fluid stream decelerates isentropically to the total pressure p_{0y} *at the entrance to the Pitot tube.*

A pitot tube traVelling in a supersonic wind-tunnel giVes Values of 16 *kPa and* 70 *kPa for the static pressure upstream of the shock and the pressure at the mouth of the tube respectiVely. Estimate the Mach number of the tunnel. If the stagnation temperature is* 300°C*, calculate the static temperature and the total* (*stagnation*) *pressure upstream and downstream of the tube.*

Solution With reference to the Fig. Ex. 17.4

$$p_x = 16 \text{ kPa}, p_{oy} = 70 \text{ kPa}$$
$$\therefore \quad \frac{p_{oy}}{p_x} = \frac{70}{16} = 4.375$$

From the gas tables for normal shock

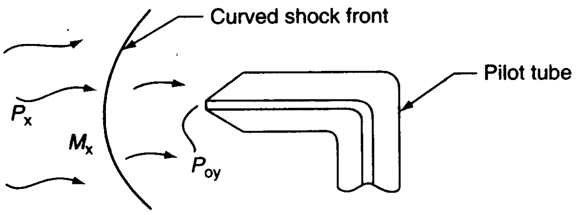


Fig. Ex. 17.4

When

$$\frac{p_{oy}}{p_x} = 4.375, M_x = 1.735, \frac{p_y}{p_x} = 3.34, \frac{\rho_y}{\rho_x} = 2.25$$
$$\frac{T_y}{T_x} = 1.483, \frac{p_{oy}}{p_{ox}} = 0.84, M_y = 0.631$$
$$T_{ox} = T_{oy} = 573 \text{ K}$$

$$T_{ox} = \left(1+\frac{\gamma-1}{2}M_x^2\right)T_x = (1 + 0.2 \times 3)T_x = 1.6\ T_x$$

$$T_x = \frac{573}{1.6} = 358\ \text{K}$$

$$T_y = 358 \times 1.483 = 530\ \text{K} = 257°\text{C}$$

$$p_{ox} = \frac{p_{oy}}{0.84} = \frac{70}{0.84} = 83.3\ \text{kPa}$$

$$M_x = 1.735$$

Ans.

Example 17.5

A conVergent-diVergent nozzle operates at off-design condition while conducting air from a high pressure tank to a large container. A normal shock occurs in the diVergent part of the nozzle at a section where the cross-sectional area is 18.75 *cm*^{2}*. If the stagnation pressure and stagnation temperature at the inlet of the nozzle are* 0.21 *MPa and* 36°*C respectiVely, and the throat area is* 12.50 *cm*^{2} *and the exit area is* 25 *cm*^{2} *estimate the exit Mach number, exit pressure, loss in stagnation pressure, and entropy increase, during the flow between the tanks.*

Solution With reference to Fig. Ex. 17.5, at shock section

$$\frac{A_x}{A^*} = \frac{18.75}{12.50} = 1.5$$

Upto the shock, the flow is isentropic. From the isentropic flow tables, when

$$\frac{A}{A^*} = 1.5, M_x = 1.86$$

$$\frac{p_x}{p_{ox}} = 0.159$$

∴ Static pressure upstream of the shock

$$p_x = 0.159 \times 0.21 \times 10^3 = 33.4\ \text{kPa}$$

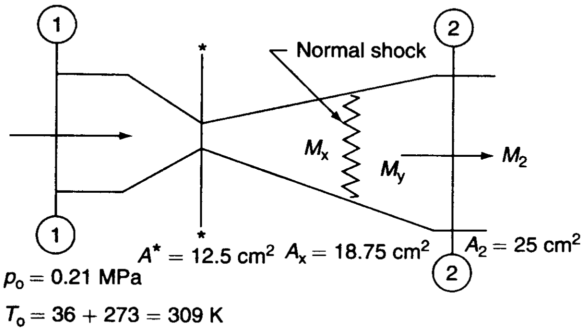


Fig. Ex. 17.5

From the gas tables on normal shocks

When $M_x = 1.86, M_y = 0.604, \frac{p_y}{p_x} = 3.87, \frac{p_{oy}}{p_x} = 4.95, \frac{p_{oy}}{p_{ox}} = 0.786$

∴ $p_y = 3.87 \times 33.4 = 129.3$ kPa

$p_{oy} = 4.95 \times 33.4 = 165.3$ kPa

From the shock section to the exit of the nozzle, the flow is again isentropic.

When $M_y = 0.604$, from the isentropic flow tables

$$\frac{A_y}{A^*} = 1.183$$

$$\therefore \quad \frac{A_2}{A^*} = \frac{A_2 A_y}{A_y A^*} = \frac{25}{18.75} \times 1.183 = 1.582$$

When $A_2/A^* = 1.582$, from the isentropic flow tables, $M_2 = 0.402$ *Ans.*

$$\frac{p_2}{p_{oy}} = 0.895$$

$$\therefore \quad p_2 = 0.895 \times 165.3 = 147.94 \text{ kPa}$$ *Ans.*

Loss in stagnation pressure occurs only across the shock

$$p_{ox} - p_{oy} = 210 - 165.3 = 44.7 \text{ kPa}$$ *Ans.*

$$\text{Entropy increase, } s_y - s_x = -R \ln \frac{p_{oy}}{p_{ox}} = 0.287 \ln \frac{210}{165.3}$$

$$= 0.287 \times 0.239 = 0.0686 \text{ kJ/kg K}$$ *Ans.*

Review Questions

17.1 What is a compressible fluid?

17.2 What are the basic laws in compressible flow?

17.3 How is sonic velocity defined in terms of pressure and density of the fluid?

17.4 Show that the sonic velocity in an ideal gas depends on the temperature and the nature of the gas.

17.5 What is Mach number?

17.6 What is a stagnation state? What do you mean by stagnation properties?

17.7 What are a nozzle and a diffuser?

17.8 Explain the effect of area change in subsonic and supersonic flows.

17.9 What do you understand by choking in nozzle flows?

17.10 Show that the discharge through a nozzle is maximum when there is a sonic condition at its throat.

17.11 What do you understand by critical pressure ratio? What is its value for air?

17.12 Explain the effect of area ratio as a function of Mach number in an isentropic nozzle?

17.13 What is M^*?

17.14 What is a shock? Where does it occur in a nozzle?

17.15 What is the impulse function?

17.16 What is a Fanno line? Why do the end states of a normal shock lie on the Fanno line?

17.17 What is a Rayleigh line? Why do the end states of a normal shock also lie on the Rayleigh line?

17.18 Where does the local sound velocity occur on the Fanno line and on the Rayleigh line?

17.19 How is the strength of a shock defined?

17.20 Explain the occurrence of choking for adiabatic flow with friction and for diabatic flow without friction.

Problems

17.1 Air in a reservoir has a temperature of 27°C and a pressure of 0.8 MPa. The air is allowed to escape through a channel at a rate of 2.5 kg/s. Assuming that the air velocity in the reservoir is negligible and that the flow through the channel is isentropic, find the Mach number, the velocity, and the area at a section in the channel where the static pressure is 0.6 MPa.

17.2 A supersonic wind tunnel nozzle is to be designed for $M = 2$, with a throat section, 0.11 m^2 in area. The supply pressure and temperature at the nozzle inlet, where the velocity is negligible, are 70 kPa and 37°C respectively. Compute the mass flow rate, the exit area, and the fluid properties at the throat and exit. Take $\gamma = 1.4$.

17.3 An ideal gas flows into a convergent nozzle at a pressure of 0.565 MPa, a temperature of 280°C, and negligible velocity. After reversible adiabatic expansion in the nozzle the gas flows directly into a large vessel. The gas in the vessel may be maintained at any specified state while the nozzle supply state is held constant. The exit area of the nozzle is 500 mm^2. For this gas $\gamma = 1.3$ and $c_p = 1.172$ kJ/kg K. Determine (a) the pressure of the gas leaving the nozzle when its temperature is 225°C, and (b) the gas mass flow rate when the pressure in the vessel is 0.21 MPa. *Ans.* 0.36 MPa, 0.48 kg/s

17.4 Air flows adiabatically through a pipe with a constant area. At point 1, the stagnation pressure is 0.35 MPa and the Mach number is 0.4. Further downstream the stagnation pressure is found to be 0.25 MPa. What is the Mach number at the second point for subsonic flow?

17.5 The intake duct to an axial flow air compressor has a diameter of 0.3 m and compresses air at 10 kg/s. The static pressure inside the duct is 67 kPa and the stagnation temperature is 40°C. Calculate the Mach number in the duct. *Ans.* 0.526

17.6 Show that for an ideal gas the fractional change in pressure across a small pressure pulse is given by

$$\frac{dp}{p} = \gamma \frac{dV}{c}$$

and that the fractional change in absolute temperature is given by

$$\frac{dT}{T} = (\gamma - 1) \frac{dV}{c}$$

17.7 An airplane flies at an altitude of 13,000 m (temperature – 55°C, pressure 18.5 kPa) with a speed of 180 m/s. Neglecting frictional effects, calculate (a) the critical velocity of the air relative to the aircraft, and (b) the maximum possible velocity of the air relative to the aircraft.

17.8 A stream of air flowing in a duct is at a pressure of 150 kPa, has a Mach number of 0.6, and flows at a rate of 0.25 kg/s. The cross-sectional area of the duct is 625 mm^2. (a) Compute the stagnation temperature of the stream in °C. (b) What is the maximum percentage reduction in area which could be introduced without reducing the flow rate of the stream? (c) For the maximum area reduction of part (b), find the velocity and pressure at the minimum area, assuming no friction and heat transfer.

17.9 A rocket combustion chamber is supplied with 12 kg/s of hydrogen and 38 kg/s of oxygen. Before entering the nozzle all the oxygen is consumed, the pressure is 2.3 MPa, and the temperature is 2800°C. Neglecting dissociation and friction, find the throat area of the nozzle required. Assume $\gamma = 1.25$.

17.10 A gas with specific heat ratio $\gamma = 1.4$ passes through a plane normal shock Immediately before the shock the Mach number is 2, and the stagnation pressure and temperature are 600 kPa and 400 K respectively. Find the static pressure and static temperature immediately downstream of the shock. *Ans.* 345 kPa, 376 K

17.11 Air flowing through a nozzle encounters a shock. The Mach number upstream of the shock is 1.8, and the static temperature downstream of the shock is 450 K. How much has the velocity changed across the shock? *Ans.* 378 m/s

17.12 The stagnation temperature and stagnation pressure of air in a reservoir supplying a convergent-divergent nozzle are 450 K and 400 kPa respectively. The nozzle throat area is 625 mm^2 and the nozzle exit area is 1875 mm^2. A shock is noted at a position in the divergent section where the area is 1250 mm^2. (a)What are the exit pressure, temperature, and velocity? (b) What value of the back pressure would cause the flow through the nozzle to be completely supersonic? (c) What value of back pressure would result in completely isentropic flow interior and exterior to the nozzle? *Ans.* (a) 234 kPa, 440 K, 141 m/s, (b) 150 kPa, (c) 18.8 kPa

17.13 Assume that a flow through the nozzle has been established by setting the back pressure at 200 kPa. Because a back pressure greater than 150 kPa leads to a shock in the nozzle, find the area ratio where the shock occurs.

17.14 Air flows through a frictionless convergent-divergent nozzle. The area of the exit section is three times the area of the throat section, and the ratio of stagnation pressure at the entrance to the static pressure in the exit section is 2.5. Calculate (a) the Mach number M_x where the shock occurs, (b) the area A_x where the shock occurs, and (c) the entropy increase.

17.15 A nozzle is designed assuming reversible adiabatic flow with an exit Mach number of 2.6. Air flows through it with a stagnation pressure and temperature of 2 MPa and 150°C respectively. The mass rate of flow is 5 kg/s. (a) Determine the exit pressure, temperature, area, and throat area. (b) If the back pressure at the nozzle exit is raised to 1.35 MPa, and the flow remains isentropic except for a normal shock wave, determine the exit Mach number and temperature, and the mass flow rate through the nozzle.

17.16 A jet plane travels through the air with a speed of 1000 km per hour at an altitude of 6000 m, where the pressure is 40 kPa and the temperature is – 12°C. Consider the diffuser of the engine. The air leaves the diffuser with a velocity of 100 m/s. Determine the pressure and temperature leaving the diffuser, and the ratio of the inlet to the exit area of the diffuser, assuming isentropic flow. *Ans.* 61 kPa, 295 K, 0.487

17.17 A normal shock occurs in the diverging section of a nozzle under steady flow conditions at a point where the air is at 150 kPa and 300 K travelling at 1000 m/s. What are the pressure and temperature on the subsonic side of the front? If the surroundings are at 20°C, what is the irreversibility caused by the shock process?

Ans. 1426.5 kPa, 762.1 K, 84.74 kJ/kg

CHAPTER

18

Elements of Heat Transfer

18.1 BASIC CONCEPTS

Energy balances by first law have been made in a variety of physical situations, say, in a feedwater heater or a cooling coil. However, no indication has been given regarding the size of the heat exchanger, for heating or cooling of a fluid. If we consider a steel block heated in a furnace, to be allowed to cool in room air, we can estimate the amount of heat lost by the block in cooling by energy balance. But how long the cooling process will take place cannot be answered by thermodynamics. It is the science of heat transfer which is concerned with the estimation of the rate at which heat is transferred, the duration of heating or cooling for a certain heat duty and the surface area required to accomplish that heat duty.

There are three modes in which heat may be transferred: (a) conduction, (b) convection and (c) radiation.

Conduction refers to the transfer of heat between two bodies or two parts of the same body through molecules which are more or less stationary. In liquids and gases conduction results from the transport of energy by molecular motion near the wall and in solids it takes place by a combination of lattice vibration and electron transport. In general, good electrical conductors are also good thermal conductors.

Convection heat transfer occurs because of the motion of a fluid past a heated surface-the faster the motion, the greater the heat transfer. The convection heat transfer is usually assumed to be proportional to the surface area in contact with the fluid and the difference in temperature of the surface and fluid. Thus,

$$Q = hA\,[T_w - T_f]$$

where h is called the convection heat transfer coefficient, which is a strong function of both fluid properties and fluid velocity (W/m^{2}K).

Radiation heat transfer is the result of electromagnetic radiation emitted by a surface because of the temperature of the surface. This differs from other forms of electromagnetic radiation such as radio, television, X-rays and γ-rays which are not related to temperature.

18.2 CONDUCTION HEAT TRANSFER

Fourier's law of heat conduction states that the rate of heat flux is linearly proportional to the temperature gradient. For one dimensional or unidirectional heat conduction,

$$\mathbf{q} \infty \frac{dt}{dx} \quad \text{or} \quad \mathbf{q} = -K\frac{dt}{dx} \tag{18.1}$$

where **q** is the rate of heat flux in W/m^2, dt/dx is the temperature gradient in x-direction, and K is the constant of proportionality which is a property of the material through which heat is being conducted and is known as *thermal conductivity*. **q** is a vector quantity. The negative sign is being used because heat flows from a high to a low temperature region and the slope dt/dx is negative.

For a finite temperature difference $(t_1 - t_2)$ across a wall of thickness x (Fig. 18.1)

$$\mathbf{q} = -K\,\frac{t_2 - t_1}{x} = K\,\frac{t_1 - t_2}{x}\ \text{W/m}^2$$

If A is the surface area normal to heat flow, then the rate of heat transfer

$$Q = \mathbf{q}\cdot A = -KA\,\frac{t_2 - t_1}{x} \quad \text{or} \quad Q = KA\,\frac{t_1 - t_2}{x}\ \text{Watts} \qquad (18.2)$$

The dimension of thermal conductivity is W/mK. Since $dt/dx = \mathbf{q}/K$ for the same $\mathbf{q}$, if K is low (i.e., for an insulator), dt/dx will be large, i.e., there will be a large temperature difference across the wall, and if K is high (i.e., for a conductor), dt/dx will be small, or there will be a small temperature difference across the wall.

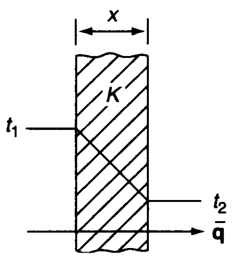


Fig. 18.1 Heat conduction through a wall

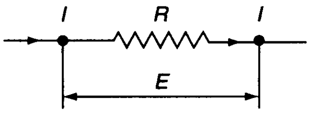


Fig. 18.2 Electrical resistance concept

18.2.1 Resistance Concept

Heat flow has an analogy in the flow of electricity. Ohm's law states that the current I flowing through a wire (Fig. 18.2) is proportional to the potential difference E, or

$$I = \frac{E}{R}$$

where $1/R$ is the constant of proportionality, and R is known as the resistance of the wire, which is a property of the material. Since the temperature difference and heat flux in conduction are similar to the potential difference and electric current respectively, the heat conduction rate through the wall may be written as

$$Q = -KA\,\frac{t_2 - t_1}{x} = \frac{t_1 - t_2}{x/KA} = \frac{t_1 - t_2}{R}$$

where $R = x/KA$ is the thermal resistance to heat flow offered by the wall (Fig. 18.3). For a composite wall, as shown in Fig. 18.4, there are two resistances in series. The slope of the temperature profile depends on the thermal conductivity of the material. t_i is the interface temperature. The total thermal resistance

$$R = R_1 + R_2 = \frac{x_1}{K_1 A} + \frac{x_2}{K_2 A}$$

and the rate of heat flow $\quad Q = \dfrac{t_1 - t_2}{R}$

Again, $t_1 - t_i = Q \cdot R_1 = Q \cdot \dfrac{x_1}{K_1 A}$, from which t_i can be evaluated.

For two resistances in parallel (Fig. 18.5), the total resistance R is given by

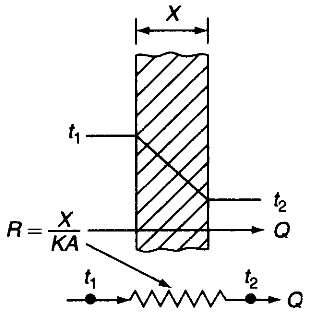


Fig. 18.3 Thermal resistance offered by a wall

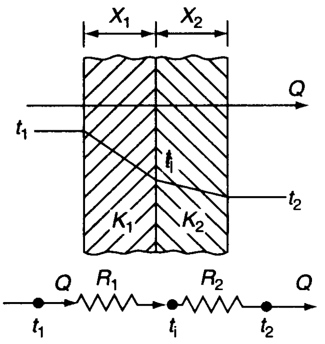


Fig. 18.4 Heat conduction through resistance in series

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

where $R_1 = \dfrac{x}{K_1 A_1}$ and $R_2 = \dfrac{x}{K_2 A_2}$

and the rate of heat flow $Q = \dfrac{t_1 - t_2}{R}$

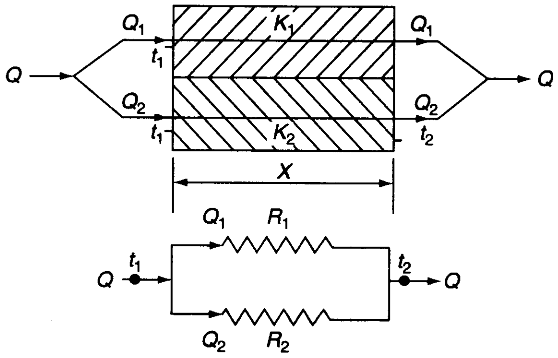


Fig. 18.5 Heat conduction through resistance in parallel

18.2.2 Heat Conduction through a Cylinder

Let us assume that the inside and outside surfaces of the cylinder (Fig. 18.6) are maintained at temperatures t_1 and t_2 respectively, and t_1 is greater than t_2. We will also assume that heat is flowing, under steady state, only in the radial direction, and there is no heat conduction along the length or the periphery of the cylinder. The rate of heat transfer through the thin cylinder of thickness dr is given by

$$Q = -KA\frac{dt}{dr} = -K\,2\,\pi rL\frac{dt}{dr} \qquad (18.3)$$

where L is the length of the cylinder.

or $$\int_{t=t_1}^{t=t_2} dt = \int_{r=r_1}^{r=r_2} -\frac{Q}{2\pi KL}\frac{dr}{r}$$

$\therefore$ $$t_2 - t_1 = -\frac{Q}{2\pi KL}\ln\frac{r_2}{r_1}$$

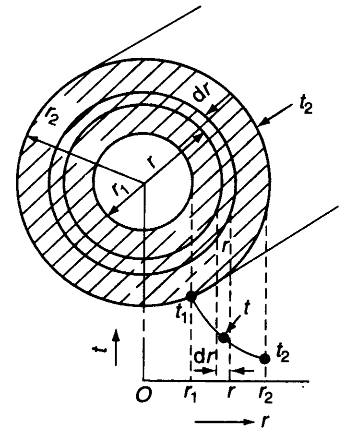


Fig. 18.6 Heat conduction through a cylindrical wall

$\therefore$ $$Q = \frac{2\pi KL(t_1 - t_2)}{\ln\dfrac{r_2}{r_1}} \qquad (18.4)$$

Equation (18.4) can also be written in the following form

$$Q = \frac{2\pi L(r_2 - r_1)K(t_1 - t_2)}{(r_2 - r_1)\ln\dfrac{2\pi r_2 L}{2\pi r_1 L}} = \frac{K(A_2 - A_1)(t_1 - t_2)}{(r_2 - r_1)\ln\dfrac{A_2}{A_1}}$$

where A_1 and A_2 are the inside and outside surface areas of the cylinder.

$\therefore$ $$Q = -KA_{\text{l.m.}}\frac{t_2 - t_1}{r_2 - r_1} \text{ or } Q = -KA_{\text{l.m.}}\frac{t_2 - t_1}{x_w} \qquad (18.5)$$

where $A_{\text{l.m.}}$ = log-mean area = $\dfrac{A_2 - A_1}{\ln\dfrac{A_2}{A_1}}$

and x_w = wall thickness of the cylinder

$$= r_2 - r_1$$

Here the thermal resistance offered by the cylinder wall to radial heat conduction is

$$R = \frac{x_w}{K \cdot A_{\mathrm{l\cdot m}}}$$

From Eq. (18.3) $$\mathrm{d}t = -\frac{Q}{2\pi KL}\frac{\mathrm{d}r}{r} = C_1 \frac{\mathrm{d}r}{r} \quad \text{or} \quad t = C_1 \ln r + C_2 \tag{18.6}$$

where C_1 and C_2 are the constants to be evaluated from the conditions: when

$$r = r_1, t = t_1 \quad r = r_2, t = t_2$$

The temperature across the wall of the cylinder thus varies logarithmically.

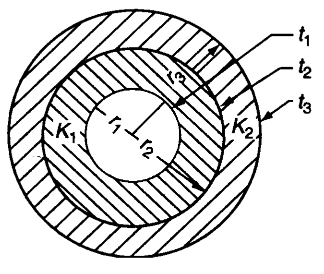
For two cylindrical resistances in series (Fig. 18.7)

$$R = R_1 + R_2 = \frac{x_{w1}}{k_1 A_{\mathrm{l\cdot m1}}} + \frac{x_{w2}}{k_2 A_{\mathrm{l\cdot m2}}}$$

where $$x_{w1} = r_2 - r_1 \quad x_{w2} = r_3 - r_2$$

$$A_{\mathrm{l\cdot m1}} = \frac{A_2 - A_1}{\ln \frac{A_2}{A_1}} = \frac{2\pi(r_2 - r_1)L}{\ln \frac{r_2}{r_1}}$$

and $$A_{\mathrm{l\cdot m2}} = \frac{A_3 - A_2}{\ln \frac{A_3}{A_2}} = \frac{2\pi(r_3 - r_2)L}{\ln \frac{r_3}{r_2}}$$



Heat conduction through two cylindrical thermal resistances in series

The rate of heat transfer will be $$Q = \frac{t_1 - t_3}{R} = \frac{t_1 - t_2}{R_1} = \frac{t_2 - t_3}{R_2}$$

from which the interface temperature t_2 can be evaluated.

18.2.3 Heat Conduction through a Sphere

Heat flowing through the thin spherical strip (Fig. 18.8) at radius r, of thickness dr, is

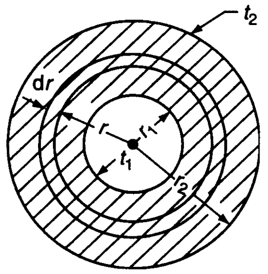
$$Q = -KA\frac{\mathrm{d}t}{\mathrm{d}r}$$

where A is the spherical surface at radius r normal to heat flow,

$$\therefore \quad Q = -K4\pi r^2 \frac{\mathrm{d}t}{\mathrm{d}r}$$

$$\int_{t=t_1}^{t=t_2} \mathrm{d}t = \int_{r=r_1}^{r=r_2} -\frac{Q}{4\pi K}\frac{\mathrm{d}r}{r^2}$$

$$t_2 - t_1 = -\frac{Q}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$



Heat conduction through a sphere

$$\therefore \qquad Q = \frac{4\pi K (t_1 - t_2) r_1 r_2}{r_2 - r_1} \quad \text{or} \quad Q = -KA_{\text{g.m.}} \frac{t_2 - t_1}{x_w} \tag{18.7}$$

where $A_{\text{g.m.}}$ = geometrical mean area

$$= \sqrt{A_1 A_2} = \sqrt{4\pi r_1^2 4\pi r_2^2} = 4\pi r_1 r_2$$

and x_w = wall thickness of the sphere = $r_2 - r_1$
Here the thermal resistance offered by the wall to heat conduction is $R = \dfrac{x_w}{KA_{\text{g.m.}}}$

Thus similar expressions of thermal resistance hold good for flat plate, cylinder, and sphere, which are

$$R_{\text{plate}} = \frac{x_w}{KA}, R_{\text{cylinder}} = \frac{x_w}{KA_{\text{l.m.}}} \quad \text{and} \quad R_{\text{sphere}} = \frac{x_w}{KA_{\text{g.m.}}}$$

where K is the thermal conductivity of the wall material.

18.2.4 Fins

Fins are often attached with heat transfer surfaces to increase the area for convection and thus increase the heat transfer rate, particularly when heat transfer coefficient is low. One type of fin is the straight fin of a rectangular profile shown in Fig. 18.9. By combining a conduction and convection energy balance on the fin and neglecting heat loss from the tip, it is possible to derive an expression for the temperature distribution at any x position as (see "Heat and Mass Transfer," by P.K. Nag, Tata McGraw-Hill, 2007)

$$\frac{t - t_\infty}{t_1 - t_\infty} = \frac{\cosh m (l - x)}{\cosh ml} \tag{18.8}$$

where t = temperature at the base of the fin, t_α = free-stream temperature, l = length of fin, $m = [(hp)/(KA)]^{1/2}$, h = heat transfer coefficient, P = perimeter of fin, A = cross-sectional area of fin and K = thermal conductivity.

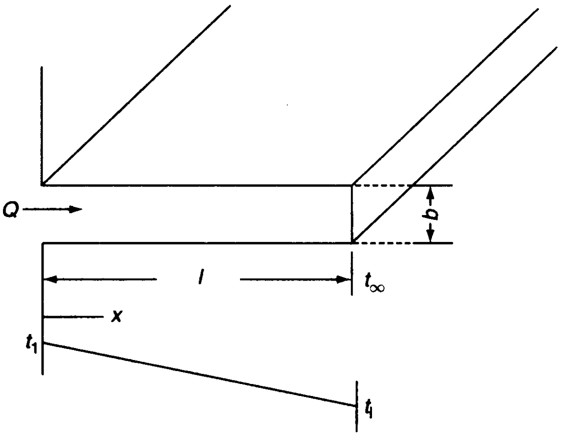


Fig. 18.9 Rectangular fin

If heat loss from the tip is neglected, the tip temperature t_l is given by:

$$\frac{t_l - t_\infty}{t_1 - t_\infty} = \frac{1}{\cosh ml}$$

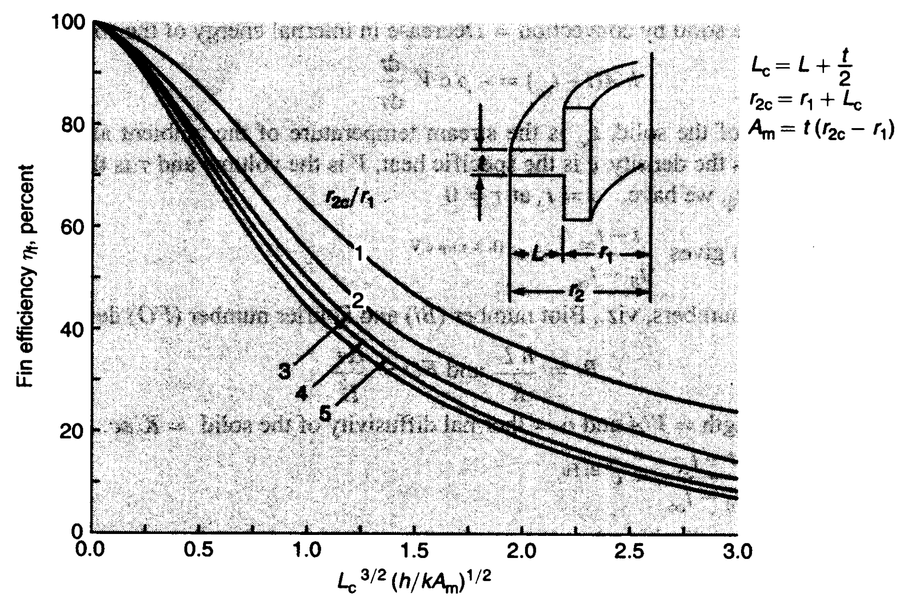
The heat transfer from one fin is

$$Q = m K A \theta_1 \tan h\, ml \tag{18.9}$$

where $\qquad \theta_I = t_1 - t_\infty.$

A fin efficiency (η_f) may be defined as:

$$\eta_f = \frac{\text{actual heat transfer}}{\text{heat which would be transferred if entire fin area were at base temperature}} \tag{18.10}$$



[illegible] *Efficiencies of circumferential fins of rectangular profile*

For the rectangular fin, the fin efficiency may be calculated from:

$$\eta_f = \frac{\tan h\, ml}{ml} \tag{18.11}$$

The hyperbolic functions are defined as;

$$\cosh x = \frac{e^{x} + e^{-x}}{2}, \tanh x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

Analytical expressions for circular fins may also be derived but are very complicated, so the results are given here in graphical form (Fig. 18.10). The top curve for $r_{cr}/r_1 = 1.0$ is also the efficiency for the straight fin of rectangular profile. From the definition of η_f, the actual heat transfer is thus:

$$Q = \eta_f\, h\, a_f(t_1 - t_\infty) + h(a - a_f)(t_1 - t_\infty) \tag{18.12}$$

where a_f = surface area of fin and a = total area of finned and unfinned surface area.

18.2.5 Transient Heat Conduction

We have so far considered only steady-state conduction where the temperatures do not vary with time. We will now consider a simple case of transient conduction, say, cooling of a solid by convection. Figure 18.11 shows a thermal "lump" which has a low internal resistance compared to the external convective resistance at the surface. Thus, there will be small temperature gradients in the solid material, so that at any instant of time the material remains essentially uniform in temperature.

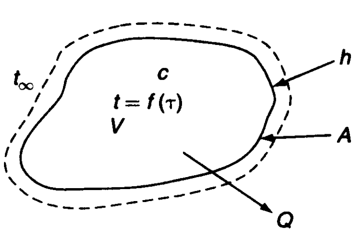


Fig. 18.11 *Cooling of a solid by convection*

Therefore, Heat lost by the solid by convection = Decrease in internal energy of the solid

$$h\,A(t - t_\infty) = -\rho\,c\,V\,\frac{dt}{d\tau} \tag{18.13}$$

where t is the temperature of the solid, t_∞ is the stream temperature of the ambient atmosphere, A is the surface area of the solid, ρ is the density, c is the specific heat, V is the volume and τ is the time.

If the solid is initially at t_0, we have, $t = t_\jmath$ at $\tau = 0$

Integration of Eq. (18.13) gives $\dfrac{t - t_\infty}{t_0 - t_\infty} = e^{-(\mathrm{h\,A}\,\tau)/\rho\,\mathrm{c\,V}}$ (18.14)

If we use dimensionless numbers, viz., Biot number (Bi) and Fourier number (FO) defined as:

$$Bi = \frac{h\,L}{K} \text{ and } FO = \frac{\alpha\tau}{L^2}$$

where L = characteristic length = V/A and α = thermal diffusivity of the solid $= K/\rho c$.

Equation (18.14) becomes $\dfrac{t - t_\infty}{t_0 - t_\infty} = e^{-\mathrm{Bi\,Fo}}$ (18.15)

This lumped-capacity is applicable when the conduction resistance is small compared to the convection resistance. In practice, this normally applies when

$$\frac{h\,L}{K} \quad \text{or} \quad Bi < 0.1 \tag{18.16}$$

Equation (18.14) can also be expressed in terms of a thermal resistance for convection, $R_{th} = 1/hA$, and a thermal capacitance, $C_{th} = \rho\,cV$, so that

$$\frac{h\,A}{\rho c V} = \frac{1}{R_{th}\,C_{th}}$$

and the system behaves as an electric capacitance discharging through a resistor.

The transient heat conduction in a lumped capacity system is thus characterized by Biot number and Fourier number. Fourier number can be regarded as dimensionless time.

18.3 CONVECTIVE HEAT TRANSFER

Convection is a process involving the mass movement of fluids. When a temperature difference produces a density difference which results in mass movement (Fig. 18.12), the process is called *free or natural convection.* Here the plate is maintained isothermal at temperature t_w, which is higher than the surrounding fluid temperature t_f. The fluid near the wall, on getting heated, moves up due to the effect of buoyancy, and is replaced by the cold fluid moving towards the wall. Thus a circulation current is set up.

When the mass motion of the fluid is caused by an external device like a pump, compressor, blower or fan, the process is called *forced convection* (Fig. 18.13). Here the fluid is made to flow along the hot surface and heat is transferred from the wall to the fluid.

Whether the convection process is natural of forced, there is always a fluid film immediately adjacent to the wall where the temperature varies from t_w to t_f (Fig. 18.14). Heat is first conducted through this fluid film and then it is transported by fluid motion.

Rate of heat transfer through the film

$$Q = -K_f \cdot A\,\frac{t_f - t_w}{\delta}$$

where δ is the film thickness and K_f is the thermal conductivity of the film.

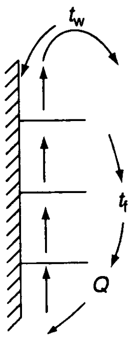


Fig. 18.12 Natural of free convection

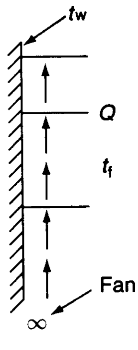


Fig. 18.13 Forced convection

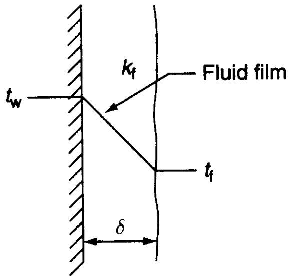


Fig. 18.14 Heat conduction through a fluid film at the wall

The film coefficient of heat transfer, or heat transfer coefficient h, is defined as

$$h = \frac{K_f}{\delta} \text{ (W/m}^2\text{ K)}$$

$$\therefore \qquad Q = hA\,(t_w - t_f) \tag{18.17}$$

The rate of heat transfer Q increases with the increase in the value of heat transfer coefficient h. The higher the value of K_f and the lower the value of film thickness δ, the higher will be the value of h. As the velocity of the fluid increases, the film thickness decreases. For gases, K_f is low, so the rate of heat transfer from the solid wall to a gas is small compared to a liquid.

The above equation is known as *Newton's law of cooling*. Strictly speaking, convection applies only to fluid motion. The mechanism of heat transfer is by conduction.

The thermal resistance offered by the fluid film

$$R = \frac{t_w - t_f}{Q} = \frac{1}{hA}$$

For heat transfer from a hot fluid to a cold fluid through a wall (Fig. 18.15), there are three resistances in series

$$R = R_1 + R_2 + R_3 = \frac{1}{h_1 A} + \frac{x}{KA} + \frac{1}{h_2 A}$$

$$Q = \frac{t_h - t_c}{R}$$

or

$$Q = \frac{t_h - t_c}{\dfrac{1}{h_1 A} + \dfrac{x}{KA} + \dfrac{1}{h_2 A}} = UA\,(t_h - t_c)$$

where U is known as the overall heat transfer coefficient (W/m^2 K) and is given by

$$\frac{1}{UA} = \frac{1}{h_1 A} + \frac{x}{KA} + \frac{1}{h_2 A} = R$$

or

$$\frac{1}{U} = \frac{1}{h_1} + \frac{x}{K} + \frac{1}{h_2}$$

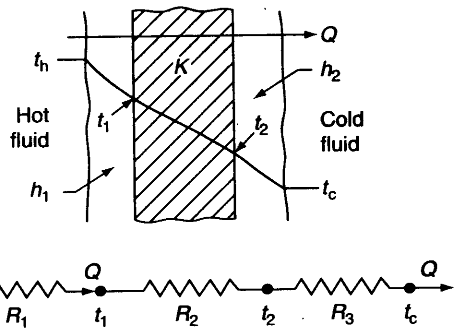


Fig. 18.15 Heat transfer from hot to cold fluid through a plane wall

For heat transfer from a hot fluid inside a cylinder to the cold fluid outside (Fig. 18.16)

$$Q = \frac{t_h - t_c}{R_1 + R_2 + R_3} = \frac{t_h - t_c}{\dfrac{1}{h_i A_i} + \dfrac{x_w}{K_w A_{l.m.}} + \dfrac{1}{h_o A_o}}$$

or $\quad Q = U_o A_o (t_h - t_c)$

where $$\frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{x_w}{K_w A_{l.m.}} + \frac{1}{h_o A_o} \qquad (18.18)$$

U_o being the overall heat transfer coefficient based on outside area A_o, h_i the inside heat transfer coefficient, and h_o the outside heat transfer coefficient.

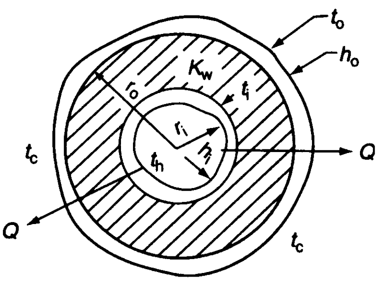


Fig. 18.16 *Radial heat transfer from hot to cold fluid through a cylinder*

When the wall thickness x_w is small, $A_o \cong A_{l.m.} \cong A_i$, and

$$\frac{1}{U_o} = \frac{1}{h_i} + \frac{x_w}{k_w} + \frac{1}{h_o}$$

18.3.1 Correlations in Convective Heat Transfer

Imagine there to be a curve in a fluid system in motion. If the tangent at every point of this curve indicates the direction of the velocity of the fluid particle, then the curve is known as a *streamline*. When one streamline slides over the other, the flow is laminar. When the streamlines are interwoven with one another and there is transverse flow of fluid particles, the flow is *turbulent*. The transition from laminar to turbulent flow in a tube depends on the mean velocity u_m, the diameter of the tube D, the density ρ, and the viscosity μ of the fluid. These variables are grouped together in a dimensionless parameter, called Reynolds number (Re), which is

$$\text{Re} = \frac{u_m D \rho}{\mu} = \frac{u_m D}{\nu}$$

where v = kinematic viscosity $\qquad = \dfrac{\mu}{\rho}$ (m^2/s)

If Re < 2100, the flow is laminar, and if Re > 2100, the flow is turbulent. The flow is most often turbulent and hardly ever laminar.

The heat transfer coefficient may be evaluated from correlations developed by dimensional analysis. In this method, all the variables pertinent to the phenomenon are to be enlisted, either by intuition or experience. Four fundamental units are selected, and these are mass M, length L, time T, and temperature θ.

Forced Convection

$$h = f(D, u, \rho, \mu, K, c_p)$$

Let $$h = B \cdot D^a \cdot u^b \cdot \rho^c \cdot \mu^d \cdot K^e \cdot c_p^f$$

Expressing the variables in terms of their dimensions

$$MT^{-3}\theta^{-1} = B \cdot L^a \cdot (LT^{-1})^b \cdot (ML^{-3})^c \cdot (ML^{-1}T^{-1})^d \cdot (MLT^{-3}\theta^{-1})^e \cdot (L^2 \cdot T^{-2}\theta^{-1})^f$$

$$= B \cdot L^{a+b-3c-d+e+2f} \cdot T^{-b-d-3e-2f} \cdot M^{c+d+e} \cdot \theta^{-e-f}$$

$\therefore \quad a + b - 3c - d + e + 2f = 0$

$$-b-d-3e-2f=-3$$
$$c+d+e=1$$
$$-e-f=-1$$

The solution of these equations gives

$$a=c-1,\ b=c,\ d=-c+f,\ e=1-f$$

$$\therefore \quad h=B\cdot D^{c-1}\cdot u^{c}\cdot \rho^{c}\cdot \mu^{-c+f}\cdot k^{1-f}\cdot c_{p}^{f}$$

$$\frac{hD}{K}=B\left(\frac{uD\rho}{\mu}\right)^{c}\cdot\left(\frac{\mu c_p}{K}\right)^{f}$$

$$Nu_d=B(Re_d)^{c}\cdot(Pr)^{f} \tag{18.19}$$

where

$$Nu_d=\text{Nusselt number}=\frac{hD}{K}$$

$$Re_d=\text{Reynolds number}=\frac{uD\rho}{\mu}$$

$$Pr=\text{Prandtl number}=\frac{\mu c_p}{K}$$

The constants B, c, and f are evaluated from experimental data. For fully developed turbulent flow inside tubes, the following equation, attributed to Dittus and Boelter, may be used

$$Nu_d=0.023\,\mathrm{Re}_d^{0.8}\,\mathrm{Pr}^{n} \tag{18.20}$$

where $n=0.4$, when the fluid is heated

$n=0.3$, when the fluid is cooled

For fully developed laminar flow in a tube, (Fig. 18.17a), $R_{ed}<2100$,

$$Nu_d=4.364 \text{ for constant wall heat flux}$$

$$Nu_d=3.66 \text{ for constant wall temperature} \tag{18.21}$$

Properties (μ, c_p, k, ρ) of the fluid are to be evaluated at the mean fluid temperature:

$$t_f=\frac{t_{f_1}+t_{f_2}}{2}$$

where t_{f_1} and t_{f_2} are the inlet and exit bulk or energy-average temperatures of the fluid.

For flow over a flat plate with constant wall temperature (Fig. 18.17b)

$$Q=hA\,(t_w-t_\infty)$$

Laminar flow, $Re_L<5\times10^{5}$

$$Nu_x=0.332\,Re_x^{1/2}\,Pr^{1/3}$$

$$\overline{Nu_L}=0.664\,Re_L^{1/2}\,Pr^{1/3} \tag{18.22}$$

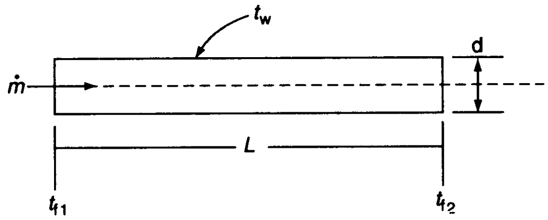


Fig. 18.17 (a) Flow in a circular tube

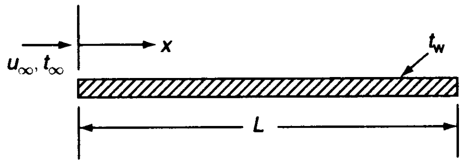


Fig. 18.17 (b) Flow over a flat plate

Turbulent flow, $Re_L > 5 \times 10^5$

$$Nu_x = 0.0296\, Re_x^{0.8}\, Pr^{1/3}$$

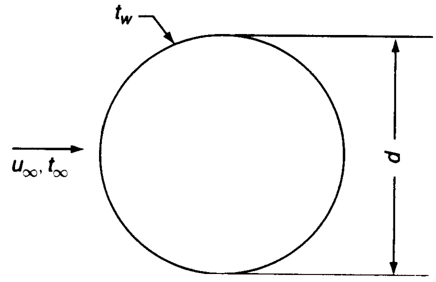
$$Nu_L = 0.037\, [Re_L^{0.8} - 850]\, Pr^{1/3} \qquad (18.23)$$

Properties are to be evaluated at $t_f = \dfrac{t_w + t_\infty}{2}$

For flow across a cylinder at constant wall temperature (Fig. 18.17c)

$$Q = h\, A\, [t_w - t_\infty]$$

$$Nu_d = C\, Re_d^n\, Pr^{1/3}$$



[illegible] (c) *Flow across a cylinder*

For flow across a sphere, constant wall temperature:

Gases: $Nu_d = 0.37\, Re_d^{0.6}$

Water and Oil: $Nu_d = [1.2 + 0.53\, Re_d^{0.54}]\, Pr^{0.3}$ (18.24)

Re_d	C	n
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40000	0.193	0.618
40000–400000	0.0266	0.805

Free Convection

Let a fluid at T_0, with density ρ_0, change to temperature T with density ρ.

Then the buoyancy force, $F = \dfrac{(\rho_0 - \rho)g}{\rho}$

Now, let β = coefficient of volume expansion

then $\dfrac{1}{\rho} = \dfrac{1}{\rho_0} + \beta(T_0 - T)$ or $\rho_0 = \rho\,(1 + \beta \cdot \Delta T)$

$\therefore$ $F = \beta \cdot g \cdot \Delta T$

where $\Delta T = T_0 - T$

For an ideal gas $\beta = \dfrac{1}{v}\left(\dfrac{\partial v}{\partial T}\right)_p = \dfrac{1}{v} \cdot \dfrac{R}{p} = \dfrac{1}{T}$ (K^{-1})

The heat transfer coefficient in free convection may be assumed to be a function of the variables as given below

$$h = f(L, K, c_p, \rho, \mu, g\,\beta\,\Delta T)$$

By dimensional analysis, the above variables may be arranged in three non-dimensional groups

$$\frac{hL}{K} = B\left(\frac{g\beta\Delta T \cdot L^3 \cdot \rho^2}{\mu^2}\right)^a \cdot \left(\frac{\mu c_p}{K}\right)^b$$

or $\mathrm{Nu} = B \cdot \mathrm{Gr}^a \cdot \mathrm{Pr}^b$

where

Nu = Nusselt number = $\dfrac{hL}{K}$

Gr = Grashoff number = $\dfrac{g\beta\Delta T L^3 \rho^2}{\mu^2}$

Pr = Prandtl number = $\dfrac{\mu c_p}{K}$

For a large number of experiments made on fluids it has been found that exponents a and b are of the same value. So the expression reduces to

$$\mathrm{Nu} = B \cdot (\mathrm{Gr} * \mathrm{Pr})^a \tag{18.25}$$

L is the characteristic length, which is the length in the case of a vertical plate and cylinder, diameter in the case of a horizontal cylinder, and radius in the case of a sphere.

For Gr. Pr $< 10^9$, the flow is laminar, and

$$\mathrm{Nu} = 0.59\,(\mathrm{Gr.\ Pr})^{1/4} \tag{18.26}$$

and for Gr. Pr $> 10^9$, the flow is turbulent, and

$$\mathrm{Nu} = 0.13\,(\mathrm{Gr.\ Pr})^{1/3} \tag{18.27}$$

18.4 HEAT EXCHANGERS

A heat exchanger is a device in which heat is transferred between two moving fluids.

Heat exchangers may be parallel flow, counterflow or crossflow, depending upon the direction of the motion of the two fluids. If both the fluids move in the same direction, it is a parallel flow heat exchanger. If the fluids flow in the opposite directions, it is a counterflow heat exchanger. If they flow normal to each other, it is a crossflow heat exchanger.

18.4.1 Parallel Flow Heat Exchanger

Let us assume that the cold fluid (subscript c) is flowing through the inner tube and the hot fluid (subscript h) is flowing through the annulus. The hot fluid enters at t_{h1} and leaves at t_{h2}, while the cold fluid enters at t_{c1} and leaves at t_{c2}. Let us consider a differential length dL of the heat exchanger, as shown in Fig. 18.18, where the hot fluid is at t_h and the cold fluid is at t_c, and the temperature difference between the two fluids is Δt $(= t_h - t_c)$. Δt varies from Δt_i, at the inlet to Δt_e at the exit of the heat exchanger. Let dQ be the rate of heat transfer in that differential length. Then by energy balance

$$dQ = U_o \cdot dA_o \cdot \Delta t = -\,\dot{m}_h c_h\, dt_h = \dot{m}_c\, c_c\, dt_c$$

Here dt_h is negative, because there is a decrease in t_h along the flow, where dt_c is positive. Symbols $\dot{m}$ and c represent the mass flow rate and specific heat respectively.

$$\Delta t = t_h - t_c$$

$$\therefore \quad d(\Delta t) = dt_h - dt_c = -\frac{dQ}{\dot{m}_h c_h} - \frac{dQ}{\dot{m}_c c_c} = -\,dQ\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right) = -\,\mu_p \cdot dQ$$

where μ_p is used for the expression in parenthesis.

$$\int_{\Delta t_i}^{\Delta t_e} d(\Delta t) = \int_i^e \mu_p\, dQ \quad \text{or} \quad \mu_p = \frac{\Delta t_i - \Delta t_e}{Q}$$

Also $$dQ = U_o\, dA_o\, \Delta t \quad \text{or} \quad -\frac{d(\Delta t)}{\mu_p} = U_o\, dA_o\, \Delta t$$

$$\therefore \quad -\int_{\Delta t_i}^{\Delta t_e} \frac{d(\Delta t)}{\Delta t} = \int_i^e \mu_p\, U_o\, dA_o$$

$$\therefore \quad \ln \frac{\Delta t_i}{\Delta t_e} = \mu_p\, U_o A_o$$

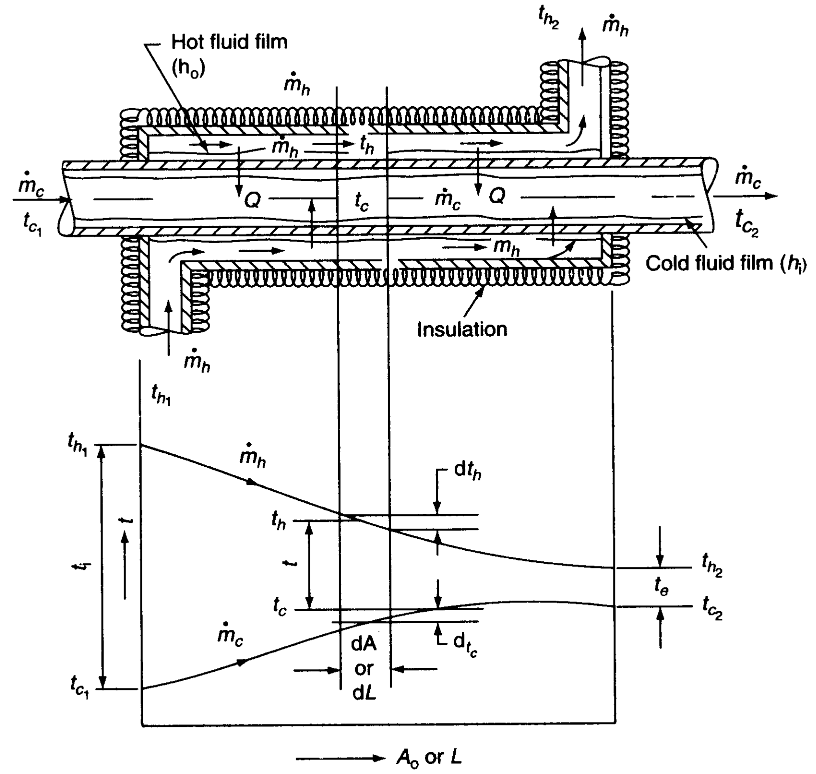


Fig. 18.18 Parallel flow heat exchanger

Substituting the expression for μ_p

$$\ln \frac{\Delta t_i}{\Delta t_e} = \frac{\Delta t_i - \Delta t_e}{Q} \cdot U_o A_o$$

$$\therefore \qquad Q = \frac{\Delta t_i - \Delta t_e}{\ln \dfrac{\Delta t_i}{\Delta t_e}} U_o A_o$$

where $\Delta t_i = t_{h1} - t_{c1}$ and $\Delta t_e = t_{h2} - t_{c2}$

$$\therefore \qquad Q = U_o A_o \Delta t_{l.m.} = \dot{m}_h c_h (t_{h1} - t_{h2}) = \dot{m}_c c_c (t_{c2} - t_{c1}) \tag{18.28}$$

where $\Delta t_{l.m.}$ = log-mean temperature difference (LMTD)

$$= \frac{\Delta t_i - \Delta t_e}{\ln \dfrac{\Delta t_i}{\Delta t_e}}$$

and
$$\frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{x_w}{k_w A_{l.m.}} + \frac{1}{h_o A_o}$$

18.4.2 Counterflow Heat Exchanger

The two fluids flow in opposite directions (Fig. 18.19). In the differential length, the rate of heat transfer

$$dQ = -\dot{m}_h c_h dt_h = -\dot{m}_c c_c dt_c = U_o \cdot dA_o \cdot \Delta t$$

where both dt_h and dt_c are negative for positive x direction (towards the right). Now

$$\Delta t = t_h - t_c$$

$$d(\Delta t) = dt_h - dt_c = -\frac{dQ}{\dot{m}_h c_h} + \frac{dQ}{\dot{m}_c c_c} = -dQ\left(-\frac{1}{\dot{m}_h c_h} - \frac{1}{\dot{m}_h c_c}\right) = -dQ\,\mu_c$$

where

$$\mu_c = \frac{1}{\dot{m}_h c_h} - \frac{1}{\dot{m}_c c_c}$$

$$\int_{\Delta t_i}^{\Delta t_e} d(\Delta t) = \int_i^e -dQ \cdot \mu_c$$

$$\Delta t_i - \Delta t_e = \mu_c \cdot Q \quad \text{or} \quad \mu_c = \frac{\Delta t_i - \Delta t_e}{Q}$$

Again

$$dQ = U_o\, dA_o\, \Delta t$$

$$-\frac{d(\Delta t)}{\mu_c} = U_o \cdot dA_o \cdot \Delta t$$

$$\int_{\Delta t_i}^{\Delta t_e} -\frac{d(\Delta t)}{\Delta t} = \int_i^e U_o\, dA_o \cdot \mu_c$$

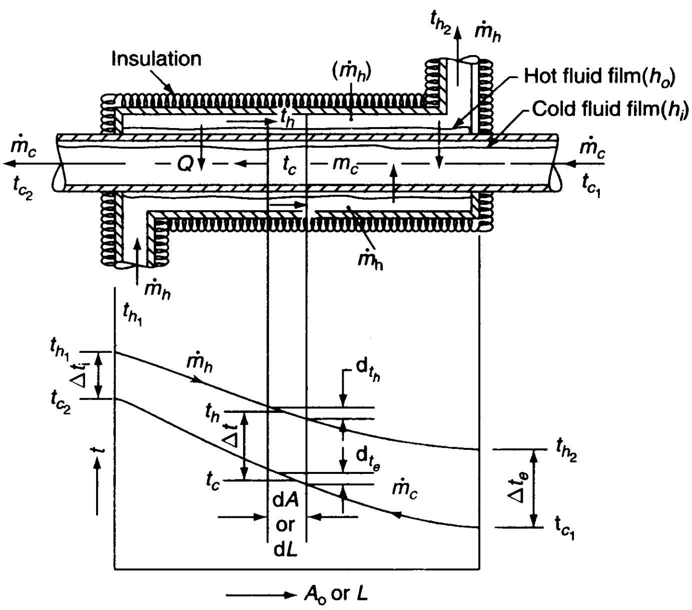


Fig. 18.19 Counterflow heat exchanger

$$\ln \frac{\Delta t_i}{\Delta t_e} = U_o \cdot A_o \cdot \mu_c = U_o A_o \cdot \frac{\Delta t_i - \Delta t_e}{Q}$$

$\therefore$

$$Q = U_o A_o \, \Delta t_{l.m.}$$

where

$$\Delta_{t\,l.m.} \cdot \frac{\Delta t_i - \Delta t_e}{\ln \dfrac{\Delta t_i}{\Delta t_e}}$$

$$\Delta t_i = t_{h1} - t_{c2}, \ \Delta t_e = t_{h2} - t_{c1}$$

$$\frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{x_w}{K_w A_{l.m.}} + \frac{1}{h_0 A_o}$$

and when x_w is small,

$$\frac{1}{U_o} \cong \frac{1}{h_i} + \frac{x_w}{K_w} + \frac{1}{h_o}$$

$\therefore$

$$Q = U_o A_o \, \Delta t_{l.m.}$$

$$= \dot{m}_c \, c_h \, (t_{h1} - t_{h2}) = \dot{m}_c c_c [t_{c1} - t_{c2}] \tag{18.29}$$

For the same rate of heat transfer, and inlet and exit temperatures, ($\Delta t_{l.m.}$) counterflow is greater than ($\Delta t_{l.m.}$) parallel flow. So the surface area required is less for counterflow operation. For parallel flow, $t_{h2} > t_{c2}$, i.e., the hot fluid cannot be cooled below t_{c2} or the cold fluid cannot be heated above t_{h2}. But for counterflow operation, t_{h2} may be less than t_{c2} which means that the hot fluid can be cooled below t_{c2} or the cold fluid heated above t_{h2}. For these reasons, counterflow heat exchangers are much more common in practice.

When one of the two fluids undergoes phase change (at constant temperature and pressure), e.g. condensation and evaporation, $\Delta t_{l.m.}$ is the same for parallel flow and the counterflow (Fig. 18.20), and the heating surface required is also the same.

18.4.3 ε-NTU Method

In a heat exchanger, the rate of heat transfer

$$Q = \dot{m}_h c_h [t_{h1} - t_{h2}] = \dot{m}_c m_c [t_{c_2} t_{c_1}] = U_o A_o \, \Delta t_{l.m.}$$

where $\dot{m}_h$, c_h, $\dot{m}_c$, c_c, t_{h_1}, t_{c_1} and U_o are usually given. Two tasks are mostly encountered:

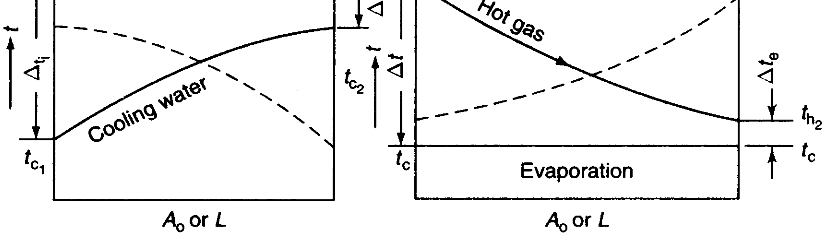


Fig. 18.20 Heat transfer with phase change

1. To estimate the surface area required (A_0). Then either t_{c_2} or t_{h_2} is given. We can use the LMTD method to find A_0.
2. For a given heat exchanger (i.e. A_0), to estimate the exit temperature t_{h_2} and t_{c_2}.

We cannot find t_{h_2} and t_{c_2} directly by the LMTD method. We have to use trial-and-error method. Assume t_{c_2}, find Q, t_{h_2}, $\Delta T_{\text{l.m.}}$ and then Q'. If $Q' \neq Q$, assume another t_{c_2} and repeat calculations till $Q' = Q$. ε-NTU method can here be easily used. We introduce three terms in this regard.

(a) Heat capacity ratio, $R = \dfrac{(\dot{m}c)_s}{(\dot{m}c)_l} = \dfrac{C_{\min}}{C_{\max}}, (< 1)$

where $(\dot{m}c)_s$ = smaller value of $\dot{m}_h c_h$ and $\dot{m}_c c_c = C_{\min}$

and $(\dot{m}c)_l$ = larger value of the two $\dot{m}_h c_h$ and $\dot{m}_c c_c = C_{\max}$

(b) Effectiveness, $\varepsilon = \dfrac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}}$

In a counterflow heat exchanger, e.g., $(t_{h_2})_{\min} = t_{c_1}$ or $(t_{c_2})_{\max} = t_{h_1}$.

$$\therefore \qquad \varepsilon = \frac{\dot{m}_h c_h \left(t_{h_1} - t_{h_2}\right)}{(\dot{m}c)_s \left(t_{h_1} - t_{c_1}\right)} = \frac{\dot{m}_c c_c \left(t_{c_2} - t_{c_1}\right)}{(\dot{m}c)_s \left(t_{h_1} - t_{c_1}\right)} (< 1)$$

If we consider $(\dot{m}c)_l$ in the denominator, then the other fluid should undergo a temperature change greater than the maximum available temperature difference, $t_{h_1} - t_{c_1}$. Therefore,

$$Q = \varepsilon \ (\dot{m}c)_s (t_{h_1} - t_{c_1}) \tag{18.30}$$

(c) NTU = Number of transfer units $= \dfrac{U_o A_o}{(\dot{m}c)_s} = \dfrac{U_o A_o}{c_{\min}}$

It gives the size of the heat exchanger. For a *parallel-flow* heat exchanger.

$$\ln \frac{\Delta t_e}{\Delta t_i} = -\mu_p U_o A_o$$

$$1 - \frac{t_{h_2} - t_{c_2}}{t_{h_1} - t_{c_1}} = 1 - e^{-\mu_p U_o A_o} = \frac{t_{h_1} - t_{c_1} - t_{h_2} + t_{c_2}}{t_{h_1} - t_{c_1}}$$

Let $\dot{m}_h c_h < \dot{m}_c c_c$. Then $R = \dfrac{\dot{m}_h c_h}{\dot{m}_c c_c} = \dfrac{t_{c_2} - t_{c_1}}{t_{h_1} - t_{h_2}}$ and $\varepsilon = \dfrac{t_{h_1} - t_{h_2}}{t_{h_1} - t_{c_1}}$

$$\varepsilon(1 + R) = 1 - \exp\left[-U_o A_o \left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right)\right]$$

$$\varepsilon_p = \frac{1 - \exp[-\text{NTU}(1 + R)]}{1 + R} \tag{18.31}$$

Similarly, for a counterflow heat exchanger,

$$\ln \frac{\Delta t_e}{\Delta t_i} = -U_o A_o \mu_c$$

$$\frac{t_{h_2} - t_{c_1}}{t_{h_1} - t_{c_2}} = e^{-U_o A_o \mu_c}$$

or, $$\frac{(t_{h_1} - t_{c_1}) - (t_{h_1} - t_{h_2})}{(t_{h_1} - t_{c_1}) - (t_{c_2} - t_{c_1})} = \frac{1 - \dfrac{t_{h_1} - t_{h_2}}{t_{h_1} - t_{c_1}}}{1 - \dfrac{t_{c_2} - t_{c_1}}{t_{h_1} - t_{c_1}}} = \frac{1 - \varepsilon}{1 - \varepsilon R} = K \text{ (say)} \quad \text{or,} \quad K - \varepsilon K R = 1 - \varepsilon$$

$$\varepsilon_c = \frac{1 - K}{1 - KR} = \frac{1 - e^{-U_o A_o \mu_c}}{1 - Re^{-U_o A_o \mu_c}}$$

$$= \frac{1 - \exp[-\text{NTU}(1 - R)]}{1 - R\exp[-\text{NTU}(1 - R)]} \tag{18.32}$$

When one of the two fluids undergoes phase change, $R = 0$. Then,

$$\varepsilon_p = \varepsilon_c = 1 - e^{-\text{NTU}} \tag{18.33}$$

For a *balanced* heat exchanger, $\dot{m}_h c_h = \dot{m}_c c_c$,

$$\therefore \quad R = 1, \Delta t_i = \Delta t_e = \Delta t_{\text{l.m.}}$$

$$t_{h_1} - t_{h_2} = t_{c_2} - t_{c_1} \text{ or, } t_{h_2} - t_{c_1} = t_{h_1} - t_{c_2}$$

$$U_o A_o \Delta t_{\text{l.m.}} = U_o A_o (t_{h_1} - t_{c_2}) = \dot{m}_h c_h (t_{h_1} - t_{h_2}) = (\dot{m}c)_s (t_{h_1} - t_{h_2})$$

$$\text{NTU} = \frac{U_o A_o}{(\dot{m}c)_s} = \frac{t_{h_1} - t_{h_2}}{t_{h_1} - t_{c_2}}$$

$$\varepsilon = \frac{t_{h_1} - t_{h_2}}{t_{h_1} - t_{c_1}} = \frac{t_{h_1} - t_{h_2}}{(t_{h_1} - t_{h_2}) + (t_{h_2} - t_{c_1})}$$

$$= \frac{\text{NTU}(t_{h_1} - t_{c_2})}{\text{NTU}(t_{h_1} - t_{c_2}) + (t_{h_1} - t_{c_2})} = \frac{\text{NTU}}{\text{NTU} + 1} \tag{18.34}$$

18.5 RADIATION HEAT TRANSFER

All bodies radiate heat. The phenomenon is identical to the emission of light. Two similar bodies isolated together in a vacuum radiate heat to each other, but the colder body will receive more heat than the hot body and thus become heated.

If Q is the total radiant energy incident upon the surface of a body some part of it (Q_a) will be absorbed, some (Q_r) will be reflected, and some (Q_t) will be transmitted through the body. Therefore,

$$Q = Q_a + Q_r + Q_t$$

or $$\frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = 1 \quad \text{or} \quad \alpha + \rho + \tau = 1$$

where α is known as *absorptivity*, ρ as *reflectivity*, and τ as transmissivity. For an *opaque* body, $\tau = 0$ and $\alpha + \rho = 1$. Most solids are opaque.

A body which absorbs all the incident radiation is called a black body. A black body is also the best radiator. Most radiating surfaces are grey and have an emissivity factor ε less than unity, where

$$\varepsilon = \frac{\text{Actual radiation of gray body at } T\,\text{K}}{\text{Radiation of a black body at } T\,\text{K}}$$

It can be shown that the *emissivity* or ability to radiate heat is equal to the *absorptivity* or ability to absorb heat (Kirchhoff's law), which justifies the statement that good absorbers are also good emitters. A brightly polished surface will have a low absorptivity and low emissivity.

The rate at which energy is radiated by a black body at temperature T (K) is given by the *Stefan-Boltzmann law*

$$Q = \sigma A T^4$$

where Q = rate of energy radiation, W

A = surface area radiating heat, m^2

and σ = Stefan-Boltzmann constant = 5.67×10^{-8} W/m^2 K^4

If the radiation from a heated body is dispersed into a spectrum by a prism, it is found that the radiant energy is distributed among various wave lengths. The *total emissive power* of a body, E, is defined as the total energy emitted by the body at a certain temperature per unit time and per unit surface area at all wavelengths. The *monochromatic emissive power*, E_λ, is defined as the radiant energy emitted by a body per unit time and per unit surface area at a particular wavelength and temperature. The variation of E_λ and λ and T is shown in Fig. 18.21. At a certain temperature,

$$E = \int_0^\infty E_\lambda \, d_\lambda \tag{18.35}$$

= Area under the curve at that temperature.

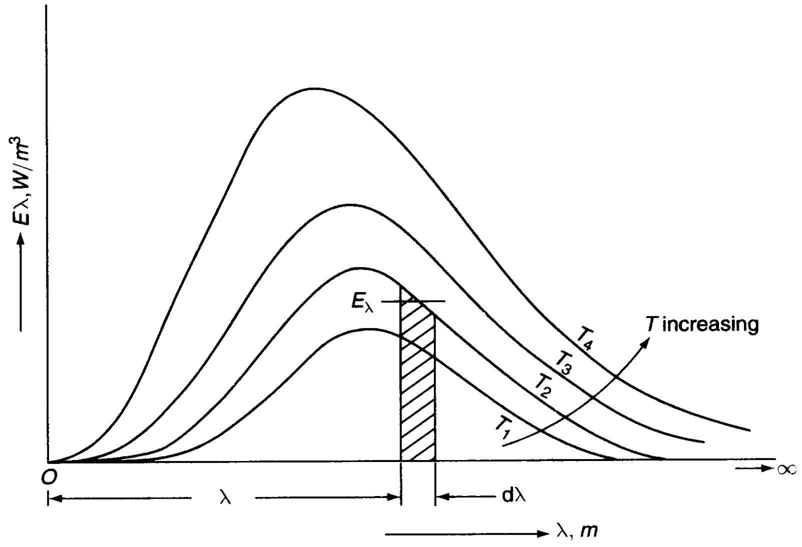


Fig. 18.21 Radiation intensity Eλ varying with λ and T

Thermal radiation extends over a spectral range of wavelengths from 0.1 μm to 100 μm and the spectral energy distribution of a black body is given by Planck's law:

$$E_{\lambda B} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1} \tag{18.36}$$

where λ = wavelength, μm; T = temperature, K;

$C_1 = 3.743 \times 10^8$ W × $(\mu\text{m})^4/\text{m}^2$; $C_2 = 1.4387 \times 10^4$ μm.K.

$E_{\lambda B}$ is called the monochromatic emissive power of a black body. The *emissitivity* of a surface is then:

$$\varepsilon = \frac{E}{E_B} \tag{18.37}$$

where E_B is the total emissive power of a black body. A *gray body* has the *monochromatic emissivity*, ε_λ, constant over all wavelengths.

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{\lambda B}} = \text{constant for a gray body} \tag{18.38}$$

Real surfaces are not gray and have a jagged emissive power distribution as shown in Fig. 18.22.

The actual radiant energy transfer between two bodies depends upon the (i) two surface temperatures, (ii) the surface emissivities, and (iii) the geometric orientation of the surfaces, i.e., how they view each other. *A radiation shape factor* F_{12} (or view factor) is defined as the fraction of energy leaving surface 1 and reaching surface 2. Similarly, F_{21} is the fraction of energy leaving 2 and reaching 1. It can be shown that

$$A_1 F_{12} = A_2 F_{21} \tag{18.39}$$

which is known as the *reciprocity theorem*.

The total emissive power of a black body is given by:

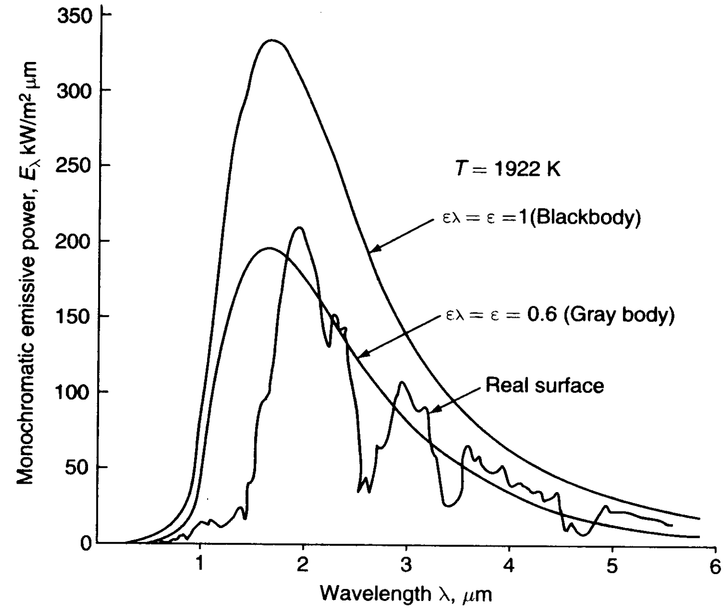


Fig. 18.22 Comparison of ideal blackbody, gray body and real surface

$$E_B = \int_0^\infty E_{\lambda B}\, d\lambda = \int_0^\infty \frac{C_1\lambda^{-5}}{e^{C_2/\lambda T} - 1}\, d\lambda = \sigma T^4$$

which is the Stefan-Boltzmann law, as stated earlier.

Charts of F_{12} for three geometries are shown in Figs 18.23, 18.24 and 18.25.

In estimating radiant heat transfer from gray surfaces, two terms will be introduced.

Radiosity (J) = total energy leaving a surface per unit area and per unit time (sum of emitted and reflected energies)

Irradiation (G) = total energy incident on a surface per unit area and per unit time.

The energy balance for the gray body (Fig. 18.26) assumed to be opaque ($\tau = 0$) gives:

$$J = \varepsilon E_B + \rho G = \varepsilon E_B + (1 - \varepsilon)G$$

$$G = [J - \varepsilon E_B]/[1 - \varepsilon]$$

The net energy leaving the surface is:

$$\frac{Q}{A} = J - G = \varepsilon E_B + (1 - \varepsilon)\, G - G$$

$$= \varepsilon(E_B - G) = \varepsilon \left[E_B - \frac{J - \varepsilon E_B}{1 - \varepsilon}\right] = \varepsilon\, \frac{E_B - J}{1 - \varepsilon}$$

$$Q_{net} = \frac{E_B - J}{(1 - \varepsilon)/A} \tag{18.40}$$

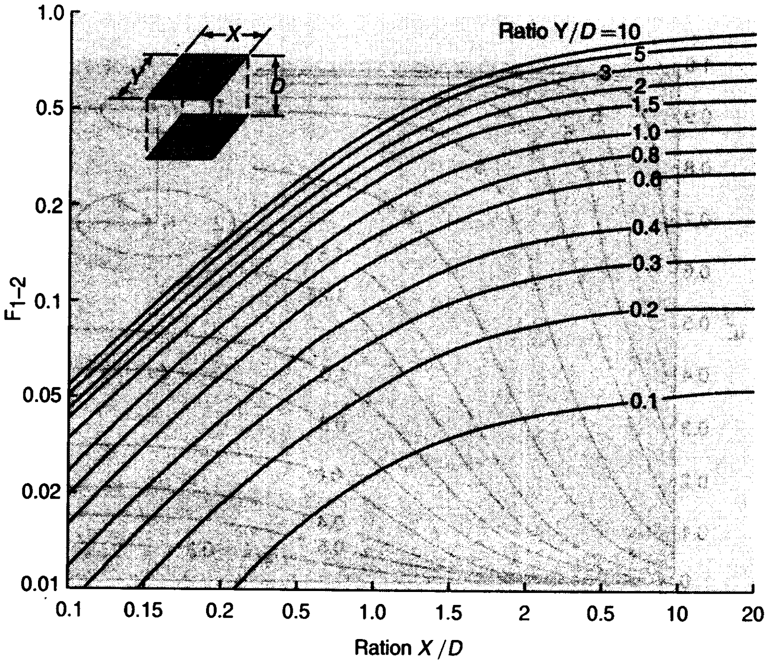


Fig. 18.24 *Radiation shape factor for radiation between parallel rectangles*

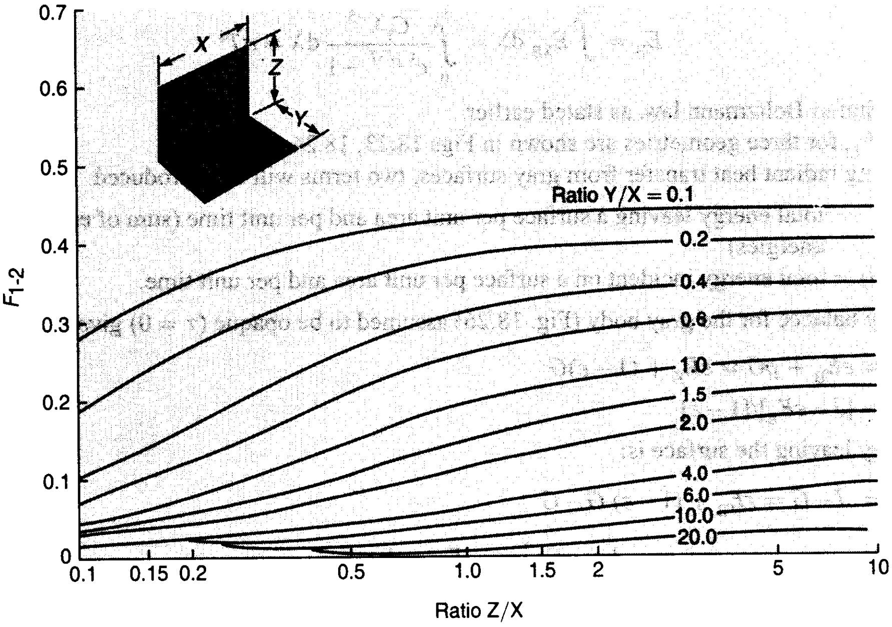


Fig. 18.24 *Radiation shape factor for radiation between perpendicular rectangles with a common edge*

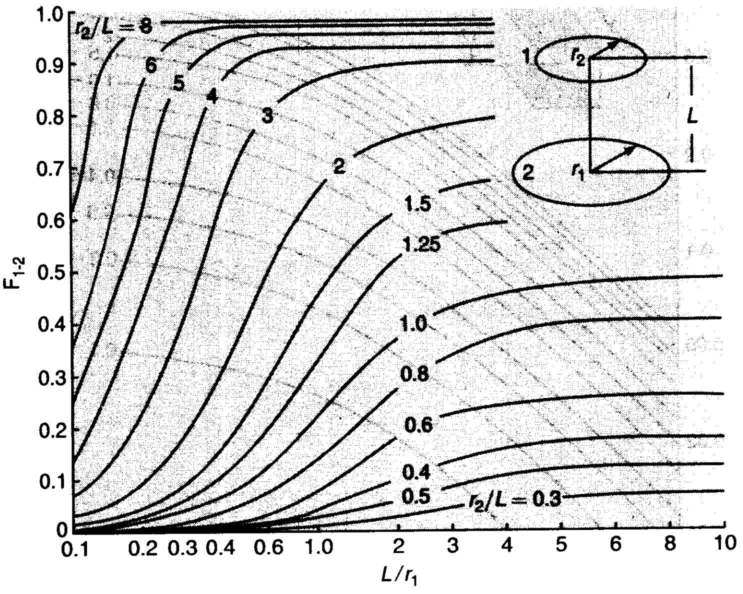


Fig. 18.25 *Radiation shape factor for radiation between two parallel concentric disks*

If the numerator is considered as the potential difference, the denominator as the "*surface resistance*" to radiation, and the heat flow as current, then a network element could be drawn as shown in Fig. 18.27.

Let us now consider the exchange of radiant energy by two surfaces A_1 and A_2 (Fig. 18.28). Of the total radiation which leaves the surface 1, the amount that reaches surface 2 is $J_1 A_1 F_{12}$, and of the total energy leaving surface 2, the amount that reaches surface 1 is $J_2 A_2 F_{21}$.

The net energy interchange between two surface is:

$$Q_{1-2} = J_1 A_1 F_{12} - J_2 E_{21} A_2$$

$$= (J_1 - J_2) A_1 F_{12} = \frac{J_1 - J_2}{1/(A_1 F_{12})} \qquad (18.41)$$

The denominator $1/(A_1 F_{12})$ is called the "space resistance" and the numerator $(J_1 - J_2)$ is the potential difference, as shown in Fig. 18.29.

Figure 18.30 shows a network which represents two surfaces exchanging radiative energy with each other.

The net heat transfer is:

$$(Q_{1-2})_{\text{net}} = \frac{E_{B_1} - E_{B_1}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$= \sigma A_1 F_{12} [T_1^4 - T_2^4] \qquad (18.42)$$

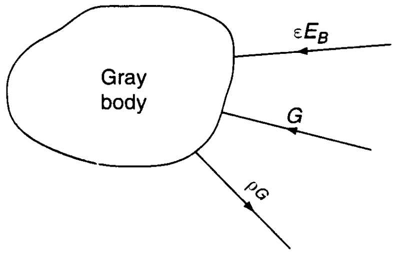


Fig. 18.26 *Radiosity and irradiation in a gray body*

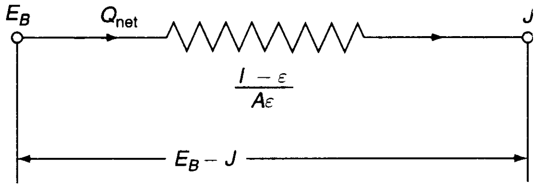


Fig. 18.27 *Surface resistance in radiation network*

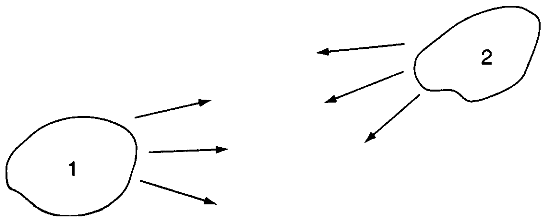


Fig. 18.28 *Radiation interaction between two gray surfaces*

where $\frac{1}{A_1 F_{12}} = \frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}$ or, $F_{12} = \dfrac{1}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{F_{12}} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$ (18.43)

For a three-body problem, the radiation network is shown in Fig. 18.31.

For two gray surfaces connected by a nonconducting and re-radiating wall in an enclosure, the radiation network is shown in Fig. 18.32.

Node J_R is not connected to any surface resistance, since the surface R has no source of its own and it only reflects energy. It is called a *floating node*. Now,

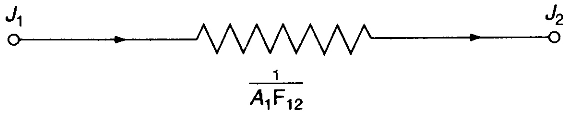


Fig. 18.29 *Space resistance in radiation network*

$$F_{11} + F_{12} + F_{1R} = 1$$

$$F_{11} = 0$$

$$F_{1R} = 1 - F_{12}$$

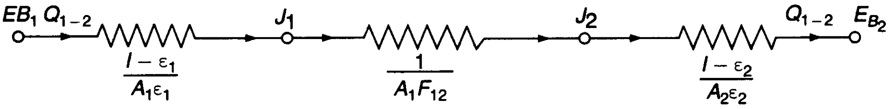


Fig. 18.30 Radiation network for two gray surfaces

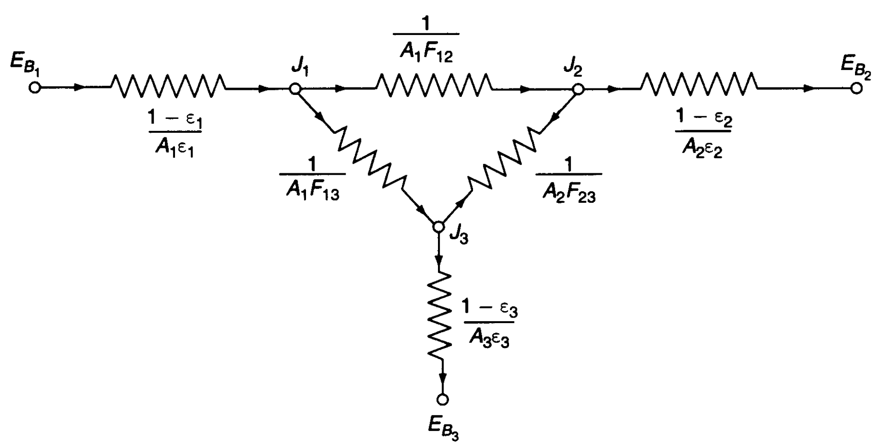


Fig. 18.31 Radiation network for three gray surfaces

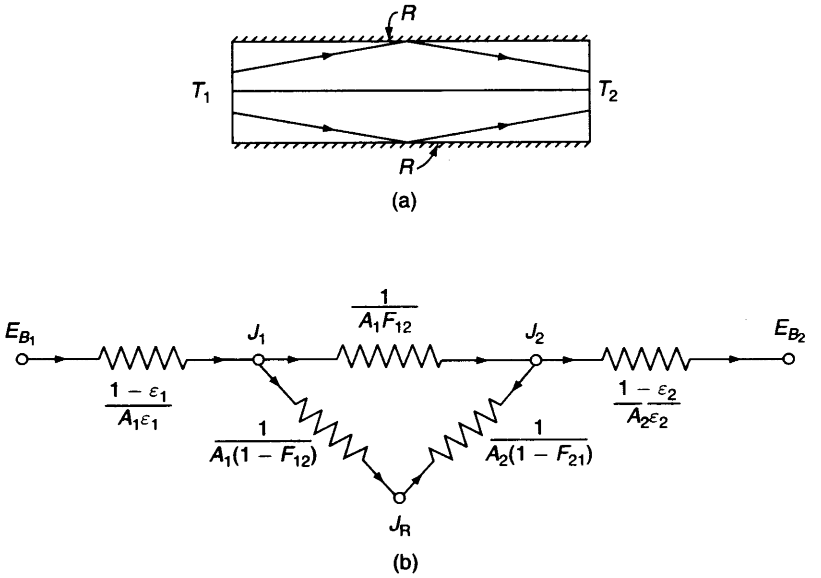


Fig. 18.32 Radiation network for two gray surfaces enclosed by a third surface (R) which is nonconducting and re-radiating

Similarly, $F_{2R} = 1 - F_{21}$

The network is a simple series-parallel system and may be solved to compute the heat flow.

From Eq. (18.43) some simple situations can be explained.

(a) For two black surfaces, $$\varepsilon_1 = \varepsilon_2 = 1, F_{12} = F_{12} \tag{18.44}$$

(b) For two infinite parallel planes, $A_1 = A_2, F_{12} = 1$

$$F_{12} = \frac{1}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} \tag{18.45}$$

(c) For two concentric cylinders or spheres, A_1 = surface area of the inner cylinder or sphere,

$$F_{12} = \frac{1}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)} \tag{18.46}$$

(d) When the enclosed body (area A_1) is very small compared to the enclosure surface, $A_2 >> A_1$, then

$$F_{12} = \varepsilon_1 \tag{18.47}$$

18.5.1 Combined Convection and Radiation

Heat is transferred from a hot body both by natural convection and radiation. Rate of heat transfer by natural convection

$$Q_c = h_c A(t_w - t_f)$$

where h_c is the convective heat transfer coefficient.

Rate of heat transfer by radiation

$$Q_r = \sigma A_1 F_{1-2} (T_w^4 - T_f^4) = h_r A_1 (t_w - t_f)$$

where h_r is known as the radiation heat transfer coefficient.

$$\therefore \quad h_r = \sigma F_{1-2} (T_w + T_f)(T_w^2 - T_f^2) \tag{18.48}$$

$\therefore$ Total rate of heat transfer $$Q = Q_c + Q_r = (h_c + h_r) A_1 (t_w - t_f) \tag{18.49}$$

Solved Examples

Example 18.1

A cold storage room has walls made of 0.23 *m of brick on the outside,* 0.08 *m of plastic foam, and finally* 1.5 *cm of wood on the inside. The outside and inside air temperatures are* 22°C *and* – 2°C *respectively. If the inside and outside heat transfer coefficients are respectively* 29 *and* 12 *W/m*^{2} *K, and the thermal conductivities of brick, foam, and wood are* 0.98, 0.02, *and* 0.17 *W/mK respectively, determine* (*a*) *the rate of heat removed by refrigeration if the total wall area is* 90 *m*^{2}*, and* (*b*) *the temperature of the inside surface of the brick.*

Solution Figure Ex. 18.1 shows the wall of the cold storage

$$\frac{1}{U} = \frac{1}{h_0} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3} + \frac{1}{h_i} = \frac{1}{12} + \frac{0.23}{0.98} + \frac{0.08}{0.02} + \frac{1.5}{100 \times 0.17} + \frac{1}{29}$$

$$\frac{1}{U} = 0.0833 + 0.2347 + 4.0 + 0.0882 + 0.0345 = 4.4407 \text{ m}^2 \text{ K/W}$$

$$\therefore \quad U = 0.2252 \text{ W/m}^2 \text{ K}$$

Rate of heat transfer $Q = UA\,(t_0 - t_i) = 0.2252 \times 90\,[22 - (-2)] = 486.4$ W *Ans.*

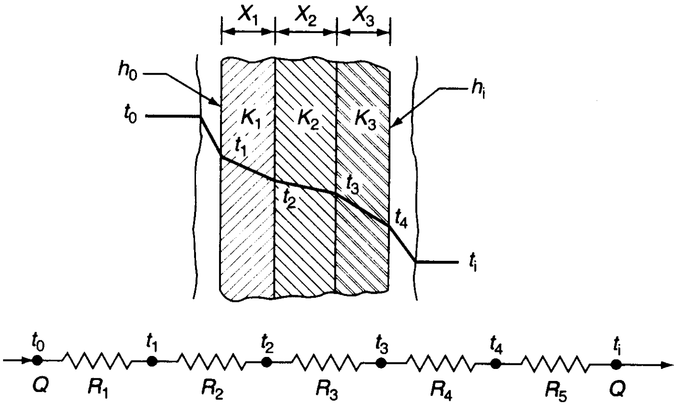


Fig. Ex. 18.1

Now

$$R_1 + R_2 = \frac{1}{h_0} + \frac{x_1}{k_1} = \frac{1}{12} + \frac{0.23}{0.98} = 0.318$$

$$\therefore \quad t_0 - t_2 = \frac{486.4 \times 0.318}{90} = 1.72°\text{C}$$

$$\therefore \quad t_2 = 22 - 1.72 = 20.28°\text{C}$$ *Ans.*

Example 18.2

Hot air at a temperature of 60°C is flowing through a steel pipe of 10 cm diameter. The pipe is covered with two layers of different insulating materials of thicknesses 5 cm and 3 cm, and their corresponding thermal conductivities are 0.23 and 0.37 W/mK. The inside and outside heat transfer coefficients are 58 and 12 W/m²K. The atmosphere is at 25°C. Find the rate of heat loss from a 50 m length of pipe. Neglect the resistance of the steel pipe.

Solution As shown in Fig. Ex. 18.2

$$r_1 = 5 \text{ cm}, \quad r_2 = 10 \text{ cm}, \quad r_3 = 13 \text{ cm}$$
$$K_1 = 0.23 \text{ and } K_2 = 0.37 \text{ W/mK}$$
$$h_i = 58 \text{ and } h_0 = 12 \text{ W/m}^2\text{/K}$$

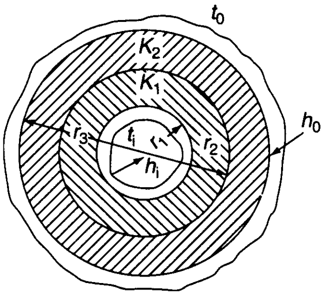


Fig. Ex.18.2

$$t_i = 60°\text{C},\ t_0 = 25°\text{C}$$

Rate of heat transfer

$$Q = \frac{2\pi L\left(t_i - t_0\right)}{\frac{1}{h_i r_i} + \frac{1}{K_1}\ln\frac{r_2}{r_1} + \frac{1}{K_2}\ln\frac{r_3}{r_2} + \frac{1}{h_0 r_3}}$$

$$= \frac{2\pi \times 50\left(60 - 25\right)}{\frac{1}{58 \times 0.05} + \frac{1}{0.23}\ln\frac{10}{5} + \frac{1}{0.37}\ln\frac{13}{10} + \frac{1}{12 \times 0.13}}$$

$$= \frac{6.28 \times 50 \times 35}{0.3448 + 0.7092 + 0.3448 + 0.6410} = 2334\ \text{W}$$

$$= 2.334\ \text{kW}$$

Ans.

Example 18.3

Three 10 mm dia. rods A, B and C protrude from a steam path at 100°C to a length of 0.25 m into the atmosphere at 20°C. The temperatures of the other ends are found to be 26.76°C for A, 32.00°C for B and 36.93°C for C. Neglecting the effects of radiation and assuming the surface film coefficient of 23 W/m² K, evaluate their thermal conductivities.

Solution If the tip loss is neglected, the tip temperature t_l is given by

$$\frac{t_l - t_\infty}{t_1 - t_\infty} = \frac{1}{\cosh ml}$$

For rod A,

$$\frac{26.76 - 20}{100 - 20} = \frac{6.76}{80} \doteq \frac{1}{\cosh ml}$$

$$\cosh ml = 11.8$$

$$\text{or,}\quad ml = 3.16$$

$$m = \frac{3.16}{0.25} = 12.64\ \text{m}^{-1} = \left[\frac{hp}{KA}\right]^{1/2}$$

Here,

$$P = \pi\,\text{d and } A = \frac{\pi}{4}d^2$$

$$\frac{23 \times \pi \times 0.01 \times 4}{K \times \pi \times 0.01 \times 0.01} = (12.64)^2$$

$$K_A = 57.58\ \text{W/mK}$$

Ans.

For rod B,

$$\frac{32 - 20}{100 - 20} = \frac{12}{80} = \frac{1}{\cosh ml}$$

$$\cosh ml = 6.67$$

$$ml = 2.6$$

or,

$$m = \frac{2.6}{0.25} = 10.4 = \left[\frac{23 \times \pi \times 0.01 \times 4}{K \times \pi \times 0.01 \times 0.01}\right]^{1/2}$$

$$K_B = \frac{9200}{(10.4)^2} = 85.2 \text{ W/mK}$$ *Ans.*

For rod C,

$$\frac{1}{\cosh ml} = \frac{36.93 - 20}{100 - 20} = \frac{16.93}{80}$$

$$\cosh ml = 4.73 \quad \text{or,} \quad ml = 2.23 \quad \text{or,} \quad m = 2.23/0.25 = 8.92$$

$$K_c = \frac{9200}{(8.92)^2} = 116 \text{ W/mK}$$ *Ans.*

Example 18.4

A 50 *mm* × 50 *mm square iron bar* 0.4 *m long is connected to the walls of two heated reservoirs, each at* 120°*C. The ambient air temperature is* 35°*C and the convective heat transfer coefficient is* 17.4 *W/m*^{2} *K. If the thermal conductivity of the bar is* 52.2 *W/mK, calculate the rate of heat loss from the bar and the temperature of the bar midway between the reservoirs.*

Solution At the central plane of the bar (Fig. Ex. 18.4) dT/dx = 0. Taking one-half of the bar,

$$\frac{\theta_l}{\theta_1} = \frac{1}{\cosh ml}, \quad \frac{Q_1}{2} = m\,K\,A\,\theta_1 \tanh m_1$$

$$\text{Here } m = \left[\frac{hp}{KA}\right]^{1/2} = \left[\frac{17.4 \times 4 \times 0.05}{52.2 \times 0.05 \times 0.05}\right]^{1/2} = 5.16 \text{ m}^{-1}$$

$$ml = 5.16 \times 0.2 = 1.032$$

$$\cosh ml = 1.58, \ \tanh ml = 0.776$$

$$\frac{t_l - 35}{120 - 35} = \frac{1}{1.58}$$

t_1 = 88.8°C = Midway temperature of the rod *Ans.*

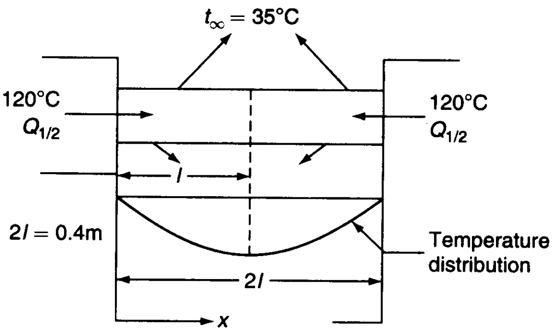


Fig. Ex. 18.4

$$Q_1/2 = 5.12 \times 52.2 \times 0.0025 \times 85 \times 0.776 = 44.2 \text{ W}$$

$$Q_1 = 88.4 \text{ W}$$ *Ans.*

Example 18.5

A load of peas at a temperature of 25°*C is to be cooled down in a room at a constant air flow of* 1°*C.* (*a*) *How long will the peas require to cool down to* 2°*C when the surface heat transfer coefficient of the peas is* 5.8 *W/m*^{2} *K?* (*b*) *What is the temperature of the peas after* 10 *min from the start of cooling?* (*c*) *What air temperature must be used if the peas were to be cooled down to* 5°*C in* 30 *min? The peas are supposed to have an average dia. of* 8 *mm. Their density is* 750 *kg/m*^{3} *and specific heat* 3.35 *kJ/kg K.*

Solution From Eq. (18.14)

$$\frac{t - t_\infty}{t_0 - t_\infty} = e^{-(hA\mathrm{t}\tau)/(\rho c V)}$$

Here,
$$\frac{\rho V}{A} = \frac{750 \times 4\pi (d/2)^3}{3 \times 4\pi (d/2)^2} = \frac{750}{3} \times \frac{d}{2} = \frac{750}{3} \times \frac{0.008}{2} = 1$$

$$\ln\frac{2-1}{25-1} = -\frac{hA\tau}{\rho c V} = -\frac{h\tau}{c} = -\frac{5.8 \times 10^{-3}\tau}{3.35}$$

(a)
$$\ln 24 = \frac{5.8 \times 10^{-3} \times 3600\,\tau}{3.35}$$

$$\tau = 0.51\ h = 31\ \text{min}$$

Ans. (a)

(b)
$$\frac{t-1}{25-1} = e^{-(5.8 \times 10^{-3} \times 10 \times 60)/3.5}$$

$$\tau = 9.5°\text{C}$$

Ans. (b)

(c)
$$\frac{5-t_\infty}{25-t_\infty} = e^{-(5.8 \times 10^{-3} \times 30 \times 60)/3.35}$$

$$\tau = 4.1°\text{C}$$

Ans. (c)

Example 18.6

An oil cooler for a lubrication system has to cool 1000 *kg/h of oil* (c_p = 2.09 *kJ/kg K*) *from* 80°*C to* 40°*C by using a cooling water flow of* 1000 *kg/h available at* 30°*C. Give your choice for a parallel flow or counterflow heat exchanger, with reasons. Estimate the surface area of the heat exchanger, if the overall heat transfer coefficient is* 24 *W/m*^{2} *K* (c_p *of water* = 4.18 *kJ/kg K*).

Solution Rate of heat transfer

$$Q = \dot{m}_h c_h (t_{h1} - t_{h2}) = \dot{m}_c c_c (t_{c2} - t_{c1})$$

$$1000 \times 2.09\,(80 - 40) = 1000 \times 4.18\,(t_{c2} - 30)$$

$$\therefore \quad t_{c2} = 50°\text{C}$$

Since $t_{c2} > t_{h2}$, counterflow arrangement must be used (Fig. Ex. 18.6).

$$(\Delta t)_{\text{l.m.}} = \frac{30-10}{\ln\frac{30}{10}} = 18.2°\text{C}$$

$$Q = 1000 \times 2.09 \times 40$$
$$= 83.6 \times 10^3 = U_o A_o \Delta t_{\text{l.m.}}$$

$$A_o = \frac{83{,}600}{24 \times 18.2 \times 3.6} = 53.16\ \text{m}^2$$

Ans.

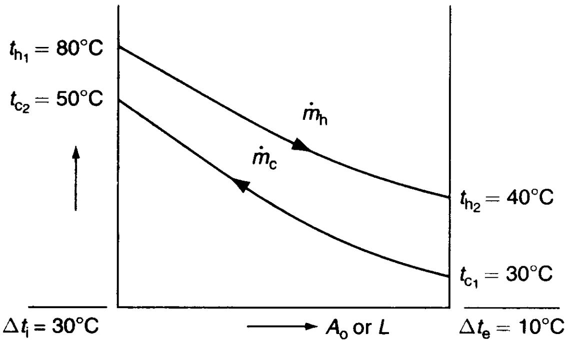


Fig. Ex. 18.6

Example 18.7

Water is evaporated continuously at 100°*C in an evaporator by cooling* 500 *kg/h of air from* 260°*C to* 150°*C. Calculate the heat transfer surface area required and the steam evaporation rate per hour if the liquid enters at* 100°*C. Take* U_0 = 46 *W/m*^{2} *K and* c_p *of air* = 1.005 *kJ/kg K.*

Solution At 100°C, $h_{fg} = 2257$ kJ/kg. If $\dot{m}$ is the rate of evaporation

$$Q = \dot{m}\, h_{fg} = \dot{m}_a\, c_{pa}\, (t_{a1} - t_{a2})$$

$$\therefore \dot{m} \times 2257 = 500 \times 1.005\,(260 - 150)$$

$$\therefore \quad \dot{m} = 24.49 \text{ kg/h} \qquad \textit{Ans.}$$

As shown in Fig. Ex. 18.7

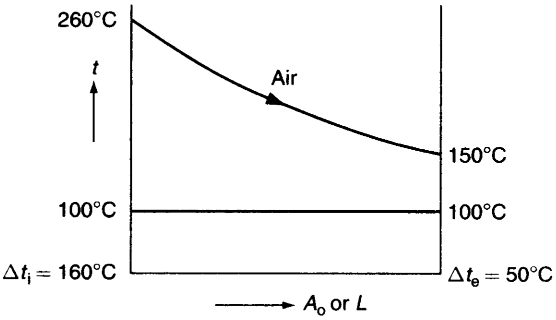


Fig. Ex. 18.7

$$\Delta t_{\text{l.m.}} = \frac{160 - 50}{\ln \dfrac{160}{50}} = 94.5\text{°C}$$

$$\therefore \quad Q = U_o A_o \Delta t_{\text{l.m.}} = 500 \times 1.005 \times 110$$

$$\therefore \quad A_o = \frac{500 \times 1.005 \times 110}{46 \times 94.5 \times 3.6} = 3.53 \text{ m}^2 \qquad \textit{Ans.}$$

Example 18.8

An oil fraction at 121°C is to be cooled at the rate of 20.15 kg/s in a simple counterflow heat exchanger using 5.04 kg/s of water initially at 10°C. The exchanger contains 200 tubes each 4.87 m long and 1.97 cm o.d., with $U_o = 0.34$ kW/m^2 K. If the specific heat of oil is 2.094 kJ/kg K, calculate the exit temperature of the oil and the rate of heat transfer.

Solution

$$\dot{m}_h = 20.15 \text{ kg/s}, c_h = 2.094 \text{ kJ/kg K}, t_{h_1} = 121\text{°C},$$

$$\dot{m}_c = 5.04 \text{ kg/s}, c_c = 4.2 \text{ kJ/kg K}, t_{c_1} = 10\text{°C},$$

$$U_o = 0.34 \text{ kW/m}^2 \text{ K}$$

$$A_o = n\, d_0\, l = 200 \times \pi \times 0.0197 \times 4.87 = 60.28 \text{ m}^2$$

$$(\dot{m}c)_{\text{oil}} = 20.15 \times 2.094 = 42.19 \text{ kW/K}$$

$$(\dot{m}c)_{\text{water}} = 5.04 \times 4.2 = 21.09 \text{ kW/K}$$

$$C_{\min} = 21.09 \text{ kW}; C_{\max} = 42.19 \text{ kW/K}$$

$$R = \frac{C_{\min}}{C_{\max}} = \frac{21.09}{42.19} = 0.5$$

$$\text{NTU} = \frac{U_0 A_0}{C_{\min}} = \frac{0.34 \times 60.28}{21.09} = 0.972$$

$$\varepsilon = \frac{1 - \exp[-\text{NTU}\,(1 - R)]}{1 - R \exp[-\text{NTU}\,(1 - R)]} = \frac{0.3849}{0.6925} = 0.5558 = \frac{\Delta t_{\text{larger}}}{t_{h_1} - t_{c_1}}$$

$$\Delta t_{\text{larger}} = (\Delta T)_{\text{water}} = 0.5558\,(121 - 10) = 61.69\text{°C}$$

$$\Delta t_{\text{oil}} = \frac{\Delta t_{\text{water}} \times (\dot{m}c)_{\text{water}}}{(\dot{m}c)_{\text{oil}}} = 30.84\text{°C} = t_{h_1} - t_{h_2}$$

$$t_{h_2} = 121 - 30.84 = 90.16\text{°C} \qquad \textit{Ans.}$$

= exit temperature of oil.

$$Q = \dot{m}_h c_h [t_{h_1} - t_{h_2}] = 42.19 \times 30.84 = 1308 \text{ kW}$$ *Ans.*

Example 18.9

Water flows inside a tube 5 *cm in diameter and* 3 *m long at a velocity* 0.8 *m/s. Determine the heat transfer coefficient and the rate of heat transfer if the mean water temperature is* 50°C *and the wall is isothermal at* 70°C. *For water at* 60°C, *take* $K = 0.66$ *W/mK,* $v = 0.478 \times 10^{-6}$ *m²/s, and* $Pr = 2.98$.

Solution Reynolds number,

$$\text{Re} = \frac{u_m D}{v} = \frac{0.8 \times 0.05}{0.478 \times 10^{-6}} = 83{,}700$$

The flow is turbulent. Prandtl number, Pr = 2.98

From the Dittus and Boelter equation

$$\text{Nu} = 0.023\,(\text{Re})^{0.8}\,(\text{Pr})^{0.4}$$

$$\frac{hD}{K} = 0.023\,(83{,}700)^{0.8}\,(2.98)^{0.4}$$

$$\frac{h \times 0.05}{0.06} = 0.023 \times 8673.6 \times 1.5478$$

$$\therefore \quad h = 4075 \text{ W/m}^2\text{ K}$$ *Ans.*

$$\therefore \quad Q = hA\,(t_w - t_f) = 4075 \times \pi \times 0.05 \times 3 \times 20 = 38387 \text{ W}$$

$$= 38.39 \text{ kW}$$ *Ans.*

Example 18.10

An electrically heated plate 15 *cm high and* 10 *cm wide is maintained at* 140°C. *Estimate the rate of heat dissipation from both sides of the plate in an atmosphere at* 20°C, *if the radiation heat transfer coefficient is* 8.72 *W/m² K. For air at the mean temperature of* 80°C, *take* $v = 21.09 \times 10^{-6}$ *m²/s,* $Pr = 0.692$, *and* $K = 0.0305$ *W/mK.*

Solution

$$Q = (h_c + h_r)\,A(t_w - t_f)$$

where $A = 0.1 \times 0.15 \times 2 = 0.03 \text{ m}^2$

$$h_r = 8.72 \text{ W/m}^2\text{ K},\; t_w - t_f = 140 - 20 = 120^\circ\text{C}$$

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = \frac{1}{T_{\text{mean}}} = \frac{1}{273 + 80} = \frac{1}{353}\;(K)^{-1}$$

$$L = 0.15 \text{ m}$$

$$\therefore \text{ Grashoff number, } \quad G_r = \frac{g\beta\,\Delta T L^3}{v^2} = \frac{9.81 \times 120 \times (0.15)^3}{353 \times (21.09 \times 10^{-6})^2} = 25.4 \times 10^6$$

$$\text{Gr. Pr} = 25.4 \times 10^6 \times 0.692 = 17.6 \times 10^6$$

Since the product Gr. Pr is less than 10^9

$$\text{Nu} = 0.59\,(\text{Gr. Pr})^{1/4} = 0.59\,(17.6 \times 10^6)^{1/4} = 38.2$$

$$\therefore \quad h_c = \frac{38.2 \times 0.0305}{0.15} = 7.77 \text{ W/m}^2\text{ K}$$

$$\therefore \quad Q = (8.72 + 7.77) \times 0.03 \times 120$$

$$= 59.36 \text{ W}$$

Ans.

Example 18.11

A long steel rod, 2 cm in diameter, is to be heated from 427°C to 538°C. It is placed concentrically in a long cylindrical furnace which has an inside diameter of 16 cm. The inner surface of the furnace is at a temperature of 1093°C, and has an emissivity of 0.85. If the surface of the rod has an emissivity of 0.6, find the time required for the heating operation. Take for steel, $\rho = 7845$ kg/m³ and $c = 0.67$ kJ/kg K.

Solution The surface area of the rod (Fig. Ex. 18.11). $A_1 = \pi \times 2 \times L$ cm², and the surface area of the furnace, $A_2 = \pi \times 16 \times L$ cm²

$$\therefore \quad \frac{A_1}{A_2} = \frac{1}{8}$$

Now $$F_{1-2} = \frac{1}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)} = \frac{1}{\dfrac{1}{0.6} + \dfrac{1}{8}\left(\dfrac{1}{0.85} - 1\right)}$$

$$= \frac{1}{1.684} = 0.592$$

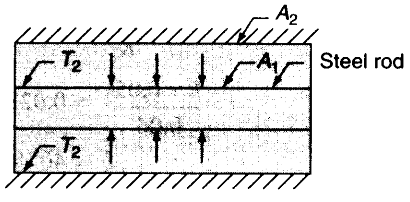


Fig. Ex. 18.11

Initial rate of heat absorption by radiation, when the rod is at 427°C or 700 K

$$Q_i = \sigma A_1 F_{1-2} (T_1^4 - T_2^4) = 5.67 \times 10^{-8} \times \pi \times 2 \times 10^{-2} \times 1 \times 0.592\,(700^4 - 1366^4) = -6332 \text{ W/m}$$

Rate of heat absorption at the end of the heating process, when the rod is at 538°C.

$$Q_e = 5.67 \times 10^{-8} \times \pi \times 2 \times 10^{-2} \times 1 \times 0.592\,(811^4 - 1366^4) = -5937 \text{ W/m}$$

$\therefore$ Average rate of heat absorption during the heating process

$$Q_{\text{a.v.}} = \frac{6322 + 5937}{2} = 6130 \text{ W/m}$$

Time required for heating, τ is obtained from the equation

$$m\, c_p\, \Delta T = Q_{\text{av}} \times \tau = 6130\, \tau$$

$$\tau = \frac{7845 \times \pi/4 \times 10^{-4} \times 1 \times 0.67 \times 111}{6130 \times 10^{-3}} \text{ s}$$

$$= 29.88 \text{ s}$$

Ans.

Example 18.12

Two long concentric cylinders exchange heat by radiation. The inner cylinder having a diameter of 10 cm has $T_1 = 1000$ K and $\varepsilon_1 = 0.65$ while the outer cylinder having a diameter of 20 cm has $T_2 = 500$ K and $\varepsilon_2 = 0.40$. Calculate the net heat transfer between the two cylinders per unit length.

Solution Referred to Fig. Ex. 18.12

$$A_1 = \pi d_1 = \pi \times 0.10 = 0.314 \text{ m}$$

$$A_2 = \pi d_2 = \pi \times 0.20 = 0.628 \text{ m}$$

$$\frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.65}{0.314 \times 0.65} = 1.715$$

$$\frac{1-\varepsilon_2}{A_2\varepsilon_2} = \frac{1-0.4}{0.628 \times 0.4} = 2.39$$

$$\frac{1}{A_2F_{12}} = \frac{1}{0.314 \times 0.4} = 3.19$$

$$\sum R = \frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}$$

$$= 1.715 + 2.39 + 3.19 = 7.295$$

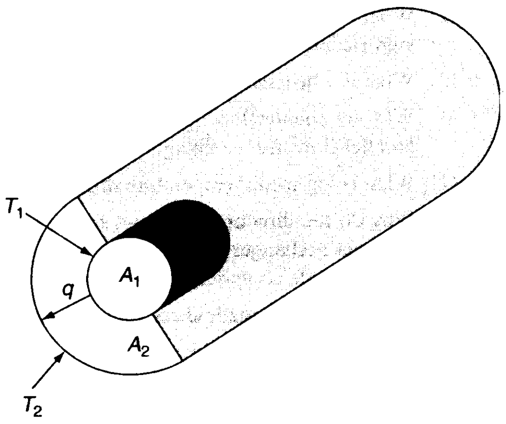


Fig. Ex. 18.12

$$E_{B_1} = \sigma T_1^4 = 5.67 \times 10^{-8} \times (1000)^4 = 5.67 \times 10^4 \text{ W/m}^2$$

$$E_{B_2} = \sigma T_2^4 = 5.67 \times 10^{-8} \times (500)^4 = \frac{5.67 \times 10^4}{16} = 0.35 \times 10^4 \text{ W/m}^2$$

$$Q = \frac{E_{B_1} - E_{B_2}}{\sum R} = \frac{5.32 \times 10^4}{7.295} = 0.729 \times 10^4 = 7290 \text{ W/m length}$$ *Ans.*

Review Questions

18.1 How is the subject of Heat Transfer different from the subject of Thermodynamics?

18.2 What are the three basic modes in which heat is transferred?

18.3 Why are good electrical conductors also good thermal conductors?

18.4 What is Fourier's law of heat conduction?

18.5 How does the slope of the temperature profile in a wall depend on its thermal conductivity?

18.6 Show that, for estimating radial heat conduction through a cylindrical wall, the log-mean area of the inner and outer surfaces has to be considered.

18.7 Show that for estimating radial heat conduction through a spherical wall, the geometric mean area of the inner and outer surfaces should be considered.

18.8 How do fins affect the heat transfer rate?

18.9 How is fin efficiency defined?

18.10 What is meant by transient heat conduction?

18.11 What is lumped-capacity analysis?

18.12 What are Biot number and Fourier number? What is their physical significance?

18.13 What do you understand by natural convection and forced convection?

18.14 How is heat transfer coefficient defined? What is its dimension?

18.15 What are the three resistances offered to heat transfer from one fluid to another through a clean wall?

18.16 What is Reynolds number? What is its critical value when the flow through a tube becomes turbulent?

18.17 What are Prandtl number and Nusselt number?

18.18 For fully developed laminar flow in a tube, what are the values of Nusselt number (a) for constant wall temperature, (b) for constant wall heat flux?

18.19 What are the expressions of Nusselt number for (a) laminar flow and (b) turbulent flow, over a flat plate?

18.20 What is Dittus-Boelter equation? Where is it used?

18.21 What is Grashof number? When does it become significant?

18.22 What is a heat exchanger?

18.23 Why are counterflow heat exchangers superior to parallel flow heat exchangers?

18.24 What is log-mean temperature difference?

18.25 Why do the directions of flows of the two fluids in a heat exchanger become immaterial when one of the two fluids undergoes phase change?

18.26 When is ε-NTU method convenient to use in heat exchanger analysis?

18.27 Define (a) effectiveness, (b) heat capacity ratio and (c) NTU, in regard to a heat exchanger.

18.28 Derive the expression for effectiveness in a (a) parallel flow heat exchanger, (b) counter-flow heat exchanger.

18.29 What is the expression for effectiveness when one of the fluids undergoes phase change?

18.30 Find the expression for effectiveness of a balanced heat exchanger with equal heat capacities.

18.31 Define absorptivity, reflectivity and transmissivity.

18.32 What is emissivity? What is Kirchhoff's law?

18.33 What is a black body?

18.34 What is the Stefan-Boltzmann law?

18.35 What is the view factor? Why is it significant in radiant heat exchange between two bodies?

18.36 What is the reciprocity theorem?

18.37 What is a gray body?

18.38 What are meant by (a) monochromatic emissive power, (b) total emissive power?

18.39 What is Planck's law of thermal radiation? Explain its importance.

18.40 Define radiosity and irradiation for a gray body.

18.41 Explain (a) surface resistance and (b) space resistance in radiant energy change between two gray bodies.

18.42 Give the radiation network for two gray surfaces and derive the view factor.

18.43 What do you mean by "floating node"?

18.44 How would you define the heat transfer coefficient for combined convection and radiation?

Problems

18.1 A room has a brick wall 25 cm in thickness. The inside air is at 25°C and the outside air is at – 15°C. The heat transfer coefficients on the inside and outside are 8.72 and 28 W/m^2 K respectively. The thermal conductivity of brick is 0.7 W/mK. Find the rate of heat transfer through the wall and the inside surface temperature.

18.2 For the wall in the above problem, it is proposed to reduce the heat transfer by fixing an insulating board ($K = 0.05$ W/mK), 2.5 cm in thickness, to the inside surface. Find the rate of heat transfer and the inside surface temperature.

18.3 Sheets of brass and steel, each 1 cm thick, are placed in contact. The outer surface of brass is kept at 100°C and the outer surface of steel is kept at 0°C. What is the temperature of the common interface? The thermal conductivities of brass and steel are in the ratio 2:1. *Ans.* 66.7°C

18.4 In a pipe carrying steam, the outside surface (15 cm OD) is at 300°C. The pipe is to be covered with insulation ($K = 0.07$ W/m K) such that the outside surface temperature does not exceed 60°C. The atmosphere is at 25°C and the heat transfer coefficient is 11.6 W/m^{2}K. Find the thickness of insulation required and the rate of heat loss per m length of pipe.

18.5 The passenger compartment of a jet transport is essentially a cylindrical tube of diameter 3 m and length 20 m. It is lined with 3 cm of insulating material (K = 0.04 W/mK), and must be maintained at 20°C for passenger comfort although the average outside temperature is – 30°C at its operating height. What rate of heating is required in the compartment, neglecting the end effects?

18.6 A hollow sphere (K = 35 W/mK), the inner and outer diameters of which are 28 cm and 32 cm respectively, is heated by means of a 20 ohm coil placed inside the sphere. Calculate the current required to keep the two surfaces at a constant temperature difference of 50°C, and calculate the rate of heat supply.

18.7 (a) Develop an expression for the steady state heat transfer rate through the walls of a spherical container of inner radius r_i and outer radius

r_0. The temperatures are t_0 and t_1 at radii r_0 and r_1 respectively. Assume that the thermal conductivity of the wall varies as

$$K = K_0 + (K_1 - K_0)\,\frac{t - t_0}{t_1 - t_0}$$

(b) Estimate the rate of evaporation of liquid oxygen from a spherical container, 1.8 m ID, covered with 30 cm of asbestos insulation. The temperatures at the inner and outer surfaces of the insulation are – 183°C and 0°C respectively. The boiling point of oxygen is – 183°C and the latent heat of vaporization is 215 kJ/kg. The thermal conductivities of the insulation are 0.16 and 0.13 W/mK at 0°C and – 183°C respectively. *Ans.* 19.8 kg/h

18.8 A counterflow double-pipe heat exchanger using superheated steam is used to heat water at the rate of 10,500 kg/h. The steam enters the heat exchanger at 180°C and leaves at 130°C. The inlet and exit temperatures of water are 30°C and 80°C respectively. If the overall heat transfer coefficient from steam to water is 814 W/m^{2}K, calculate the heat transfer area. What would the increase in area be if the fluid flows were parallel?
Ans. 7.49 m^2, 8.23 m^2, Increase = 0.74 m^2

18.9 An oil cooler consists of a straight tube of ID 1.25 cm, wall thickness 0.125 cm, enclosed within a pipe and concentric with it. The external surface of the pipe is well insulated. Oil flows through the tube at the rate of 250 kg/h and cooling water flows in the annuals at the rate of 300 kg/h in the direction opposite to that of oil. The oil enters the tube at 180°C and is cooled to 66°C. The cooling water enters at 10°C. Estimate the length of tube required, given that the heat transfer coefficient from the oil to the tube surface is 1628 and that from the tube surface to the water is 3722 W/m^2 K. Neglect the temperature drop across the tube wall, c_p of oil = 1.675 kJ/kg K. *Ans.* 2.81 m

18.10 A marine steam turbine has a condenser flow (of steam) at a full load of 12,000 kg/h. The quality of steam at the turbine exhaust is 0.85. The condenser pressure is 0.07 bar. Sea water used for circulation is at 21°C. The terminal temperature difference between the steam and outgoing water is 5.7°C. The circulating water velocity is maintained at 1.7 m/s. The condenser tubes are of 1.3 cm ID and 0.122 cm thickness. Determine (a) the rate of flow of cooling water, (b) the length of tubes, and (c) the number of tubes, in the condenser. Take U_o = 3256 W/m^2 K.
Ans. 490*t*/*h*, 6.8 m, 604

18.11 Fifty kg of water per min is heated from 30°C to 50°C by passing through a pipe of 2 cm in diameter. The pipe is heated by condensing steam on its surface maintained at 100°C. Find the length of the pipe required. Take the following properties of water at the mean bulk temperature of 40°C:
$\rho = 992.2$ kg/m^3, $v = 0.659 \times 10^{-6}$ m^2/s,
$K = 0.63$ W/mK and Pr = 4.31.

18.12 Air flows at a velocity of 0.3 m/s through the annular space between two concentric tubes. The outer tube is 5 cm ID and the inner tube is 3.125 cm OD. The air enters at 16°C and leaves at 32°C. The temperature of the outside surface of the inner tube is 50°C. Determine the heat transfer coefficient between the air and inner tube. Properties of air at 24°C: $\rho = 1.18$ kg/m^3, K = 0.03 W/mK, Pr = 0.7 and $v = 15.5 \times 10^{-6}$ m^2/s.

18.13 A rectangular duct, 30 cm × 20 cm in cross-section, carries cold air. The temperature of the outer surface of the duct is 5°C and the surrounding air temperature is 25°C. Estimate the rate of heat gain by the duct, assuming that the duct, one metre in length, is exposed to the air in the vertical position. Properties of air at 15°C: $\rho = 1.22$ kg/m^3, $v = 14.6 \times 16^{-6}$ m^2/s, K = 0.03 W/mK, and Pr = 0.7. *Ans.* 0.186 W

18.14 A hot square plate, 50 cm × 50 cm at 100°C, is exposed to atmospheric air at 20°C. Compute the rate of heat loss from both the surfaces of the plate if the plate is kept in the vertical plane. Properties of air at 60°C:

$\rho = 1.06$ kg/m^3, $v = 18.97 \times 10^{-6}$ m^2/s,

$K = 0.03$ W/mk, and Pr = 0.696

18.15 Determine the heat lost by radiation per metre length of 7.5 cm diameter oxidized steel pipe at 300°C if (a) located in a large room with red brick walls at a temperature of 25°C, and (b) enclosed in a 25 cm × 25 cm red brick conduit at a temperature of 25°C. The emissivity of oxidized steel is 0.79 and that of red brick is 0.93.
Ans. 1.052, 1.035 kW

18.16 Two concentric spheres, 21 cm and 30 cm in diameter, with the space between them evacuated are to be used to store liquid air (– 153°C) in a room at 27°C. The surfaces of the spheres are flushed with aluminium ($\varepsilon = 0.03$) and the latent heat of vaporization of liquid air is 209.35 kJ/kg. Calculate the rate of evaporation of liquid air.

Ans. 0.0216 kg/h

18.17 Estimate the net radiant heat exchange per square metre for two very large parallel planes at temperatures 560°C and 300°C respectively. Assume that the emissivities of the hot and cold planes are 0.8 and 0.6 respectively. *Ans.* 11.28 kW

18.18 The inner sphere of a Dewar flask is of 30 mm dia. and outer sphere is of 360 mm dia. Both the spheres are plated for which $\varepsilon = 0.05$. The space between them is evacuated. Determine the rate at which liquid oxygen would evaporate at – 183°C when the outer sphere temperature is 20°C. The latent heat of vaporization of liquid oxygen is 214.2 kJ/kg. [*Ans.* 1.6466×10^{-5} kg/s]

18.19 A metal plate 0.609 m in height forms the vertical wall of an oven and is at a temperature of 171°C. Within the oven is air at a temperature of 93.4°C and atmospheric pressure. Assuming that natural convection conditions hold near the plate, for which

$$\mathrm{Nu} = 0.548\,(\mathrm{Gr\,Pr})^{0.25},$$

find the mean heat transfer coefficient, and the heat taken up by the air per sec. per metre width. For air at 132.2°C, $k = 32.2 \times 10^{-6}$ kW/mK, $\mu = 0.232 \times 10^{-4}$ kg/m–s, $c_p = 1.005$ kJ/kg K. Assume air as an ideal gas with $R = 0.287$ kJ/kg K.

[*Ans.* $h = 4.18$ W/m^2 K, $Q = 197.6$ W/m]

18.20 The heat transfer coefficients for the flow of air at 28°C over a 12.5 mm dia. sphere are measured by observing the temperature-time history of a copper ball of the same dimension. The temperature of the copper ball was measured by two thermocouples, one located in the centre, the other near the surface. Both the thermocouples, measured the same temperature at an instant. In one test the initial temperature of the ball was 65°C and in 1.15 min the temperature decreased by 11°C. Calculate the heat transfer coefficient for this case.

[*Ans.* 37.41 W/m^2 K]

18.21 A cubical piece of aluminium 1 cm on a side is to be heated from 50°C to 300°C by a direct flame. How long should the aluminium remain in the flame, if the flame temperature is 800°C and the convective heat transfer coefficient is 190 W/m^2 K? For aluminium, take $\rho = 2719$ kg/m^3 and $c = 0.871$ kJ/kg K. [*Ans.* 8.36 s]

18.22 The cooling system of an electronic package has to dissipate 0.153 kW from the surface of an aluminium plate 100 mm × 150 mm. It is proposed to use 8 fins, each 150 mm long and 1 mm thick. The temperature difference between the plate and the surroundings is 50 K, the thermal conductivity of the plate and fins is 0.15 kW/mK and the convective coefficient is 0.04 kW/m^{2}K. Calculate the height of fins required. [*Ans.* 30 cm]

18.23 Oil ($c_p = 2$ kJ/kgK) is cooled from 110°C to 70°C by a flow of water in a counterflow heat exchanger. The water ($c_p = 4.18$ kJ/kgK) flows at the rate of 2 kg/s and is heated from 35°C to 65°C. The overall heat transfer coefficient is 0.37 kW/m^{2}K. Determine the exit temperatures of oil and water, if the water flow rate drops to 1.5 kg/s at the same oil flow rate. *Ans.* 72.5°C, 72.5°C

18.24 A tank contains 272 kg of oil which is stirred so that its temperature is uniform. The oil is heated by an immersed coil of pipe 2.54 cm dia. in which steam condenses at 149°C. The oil of specific heat 1.675 kJ/kgK is to be heated from 32.2°C to 121°C in 1 hour. Calculate the length of pipe in the coil if the surface coefficient is 0.653 kW/m^{2}K.

[*Ans.* 3.47 m]

CHAPTER 19

Gas Compressors

A gas compressor is a device in which work is done on the gas to raise its pressure, with an appreciable increase in its density. The compression of gases is an important process in many power plants, refrigeration plants, and industrial plants. Industrial uses occur in connection with compressed air motors for tools, air brakes for vehicles, servo-mechanisms, metallurgical and chemical processes, conveying of materials through ducts, transporting of natural gas, and production of bottled gases.

Compressors can be (a) positive displacement machines like reciprocating compressors, Root's blower and vane-sealed machines and (b) turbine type like centrifugal and axial flow compressors.

19.1 COMPRESSION PROCESSES

A gas compression process may be either adiabatic or involving heat transfer, depending upon the purpose for which the gas is compressed. If the compressed gas is to be used promptly in an engine or in a combustion process, adiabatic compression is desirable so as to obtain the maximum energy in the gas at the end of compression. In many applications, however, the gas is not used promptly but is stored in a tank or receiver for use later when needed. The gas in the tank transfer heat to the surroundings so that when finally used it is at room temperature. In such cases the objective of the compression and storage process is simply to increase the pressure of the gas without change of temperature. It will be shown below that if the gas is cooled during compression, the work required will be less than for adiabatic compression. Another advantage of cooling is the reduction in volume of gas and less pipe friction losses. Since cooling during the compression process is not very effective, *after-coolers* are often used to cool the gas leaving the compressor.

19.2 WORK OF COMPRESSION

The steady flow energy equation for a compressor (Fig. 19.1) gives

$$h_1 + Q = h_2 + W_x \tag{19.1}$$

neglecting the changes of potential and kinetic energy. From the property relation

$$T\mathrm{d}s = \mathrm{d}h - v\mathrm{d}p \tag{19.2}$$

For a reversible process,

$$Q = \Delta h - \int v\mathrm{d}p \tag{19.3}$$

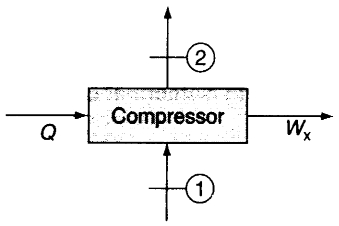


Fig. 19.1

Let us consider two particular idealized cases, namely, reversible adiabatic and reversible isothermal, as well as a general case of a reversible polytropic process (pv^n = constant). The paths of such processes are plotted in Fig. 19.2 for an ideal gas compressed from state 1 to state 2.

As shown in Sec. 7.14 or from Eqs (19.1 to 19.3) for adiabatic compression of unit mass,

$$W_x = -\int v\mathrm{d}p \tag{19.4}$$

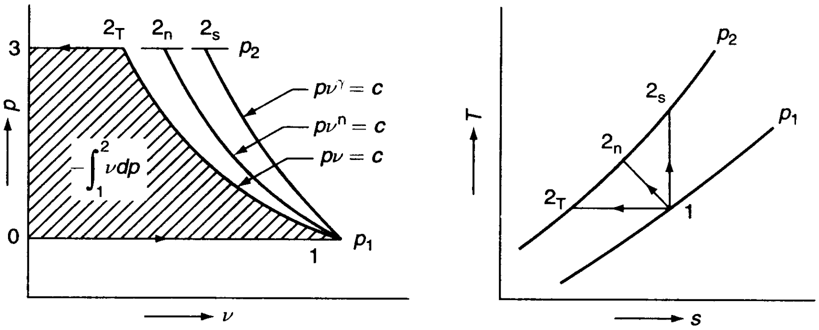


Fig. 19.2 Reversible compression processes

For a reversible adiabatic process, $pv^\gamma = C$,

$$v = \left(\frac{pv^\gamma}{p}\right)^{\frac{1}{\gamma}} = \frac{p_1^{1/\gamma} v_1}{p^{1/\gamma}} = \frac{p_2^{1/\gamma} v_2}{p^{1/\gamma}}$$

and

$$W_x = -\int_1^2 \frac{p_1^{1/\gamma} v_1}{p^{1/\gamma}} dp = -p_1^{1/\gamma} \cdot v_1 \cdot \frac{1}{1-\frac{1}{\gamma}} \left[p_2^{1-\frac{1}{\gamma}} - p_1^{1-\frac{1}{\gamma}} \right]$$

$$= -\frac{\gamma}{\gamma - 1} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

The *work of compression* or the steady flow work input to the gas is the negative of the shaft work W_x, i.e. for reversible adiabatic compression,

$$W_s = \frac{\gamma}{\gamma - 1} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \tag{19.5}$$

Similarly, for reversible polytropic compression

$$W_n = \frac{n}{n - 1} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] \tag{19.6}$$

For reversible isothermal compression of an ideal gas,

$$W_t = -\int_1^2 v dp = -\int_1^2 \frac{p_1 v_1}{p} dp = p_1 v_1 \ln \frac{p_2}{p_1} \tag{19.7}$$

In the p-v plot of Fig. 19.2 the work of compression for each type of process is represented by the area between the path of that process and the axis of pressures. For the general process $pv^n = C$, taking logarithm

$$\ln p + n \ln v = \ln C$$

On differentiation,

$$\frac{dp}{p} + n\frac{dv}{v} = 0$$

∴ The slope at any state (say 1) is given by

$$\frac{dp}{dv} = -n\frac{p_1}{v_1} \tag{19.8}$$

For $\gamma > n > 1$ and for the same pressure ratio p_2/p_1, the isothermal compression needs the minimum work, whereas adiabatic compression needs the maximum work, while the polytropic compression needing work in between the two. In isothermal compression all the work done on the gas during the process is transformed into an internal energy increase where $Q - W = u_2 - u_1 = O$ (for an ideal gas) or $W = Q$. This heat is taken away from the gas by cooling. Thus in isothermal compression, considered to be ideal, no energy would be imparted to the gas, since its function is simply to raise the pressure of the gas (and not its temperature).

The efficiency of a compressor working in a steady flow process may be defined as

$$\eta_c = \frac{h_{2S} - h_1}{W_c} = \frac{W_s}{W_c} \tag{19.9}$$

where W_c is the shaft work supplied to the compressor per unit mass. For the idealized reversible isothermal process the compressor efficiency is sometimes defined as

$$\eta_c = \frac{W_t}{W_c} \tag{19.10}$$

where W_t is the work of reversible isothermal compression. The two efficiencies of Eqs (19.9) and (19.10) are called respectively the *adiabatic efficiency* and *isothermal efficiency*. Because of the effects of cooling, the adiabatic efficiency of a real compressor may be greater than unity.

Many turbine-type compressors are essentially adiabatic machines due to their high speeds. For an adiabatic machine the work of compression is equal to the enthalpy rise of the gas,

$$W_c = h_2 - h_1$$

Then for an adiabatic compressor the efficiency is

$$\eta_c = \frac{h_{2S} - h_1}{h_2 - h_1} \tag{19.11}$$

For a reciprocating machine the compressor efficiency may be on an indicated work basis or a brake work basis, depending upon where the work input is measured (See Sec. 3.3).

19.3 SINGLE-STAGE RECIPROCATING AIR COMPRESSOR

Figure 19.3 shows the arrangement of a single-stage reciprocating air compressor, together with a typical indicator diagram. The compressor operates on a two-stroke cycle as follows:

Stroke 1 (a-c) The piston withdraws, causing the air in the clearance volume to expand, and when the pressure in the cylinder falls below atmospheric pressure (at *b*) the inlet valve opens and air is drawn into the cylinder for the remainder of the stroke.

Stroke 2 (c-a) The piston moves inwards, compressing the air in the cylinder, and the inlet valve closes when the cylinder pressure reaches atmospheric pressure. Further compression follows as the piston moves towards the top of its stroke until, when the pressure in the cylinder is more than that in the receiver, the delivery valve opens and air is delivered to the receiver for the remainder of the stroke.

The intercept *V* on the indicator diagram represents the volume of air taken in per cycle. It is seen that the effect of clearance air is to reduce the quantity of air drawn in during the suction stroke, so that in practice the clearance space is made as small as possible.

The total area of the indicator diagram represents the actual work of the compressor on the gas or air. The areas above p_2 and below p_1 represent work done because of pressure drop through the valves and port

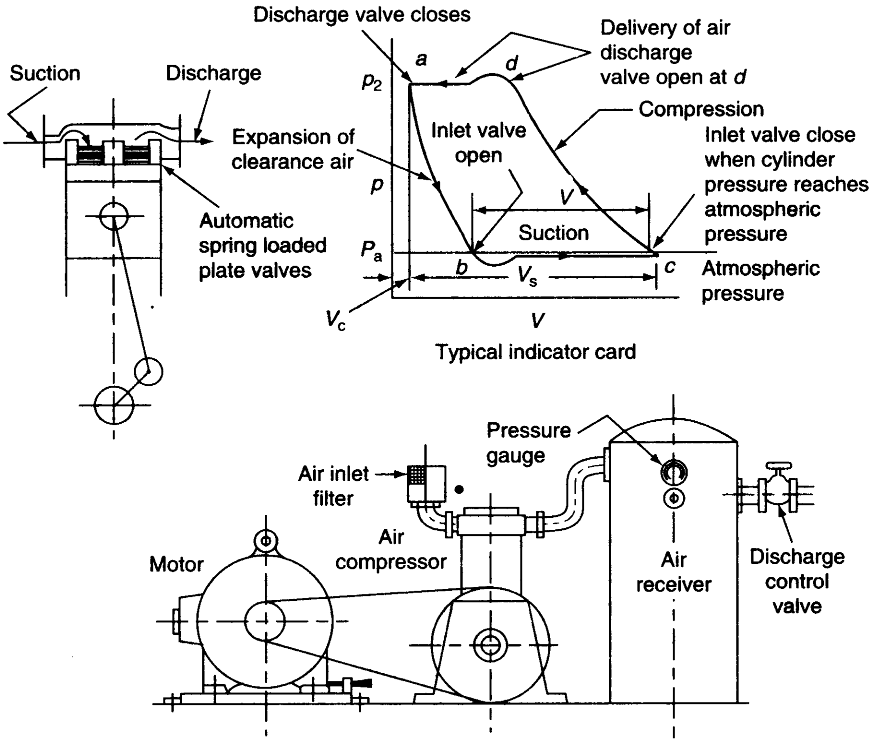


Fig. 19.3 *Arrangement of single-stage reciprocating air compression*

passages; this work is called the valve loss. The idealized machine to which the actual machine is compared has an indicator diagram like Fig. 19.4 Both expansion and compression are supposed to follow the same law $pv^n = C$. The small quantity of high pressure air in the clearance volume expands to V_a and air drawn in during the suction stroke is $V_b - V_a$. From the diagram:

Work done on the air during the cycle = Enclosed area *abcd* = Area *dcfh* + Area *cbef* − Area *hgad* − Area *geba*

$$\therefore \quad W = p_2(V_c - V_d) + \frac{p_2V_c - p_1V_b}{n-1} - \frac{p_2V_d - p_1V_a}{n-1} - p_1(V_b - V_a)$$

$$= \frac{n}{n-1}\left[(p_2V_c - p_1V_b) - (p_2V_d - p_1V_a)\right]$$

Now, $p_2 V_c = m_c R T_2, p_1 V_b = m_b RT_1$ and $m_c - m_b$ = mass of air taken during compression.

Also, $p_2 V_d = m_d RT_2, p_1 V_a = m_a RT_1$ and $m_a - m_d$ = mass of air present in the clearance volume after delivery.

$$\therefore \quad \text{Work done/cycle} = \frac{n}{n-1} R[m_b(T_2 - T_1) - m_d(T_2 - T_1)]$$

$$= \frac{n}{n-1}(m_b - m_d)R(T_2 - T_1)$$

$$= \frac{n}{n-1}(m_b - m_d)RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \qquad (19.12)$$

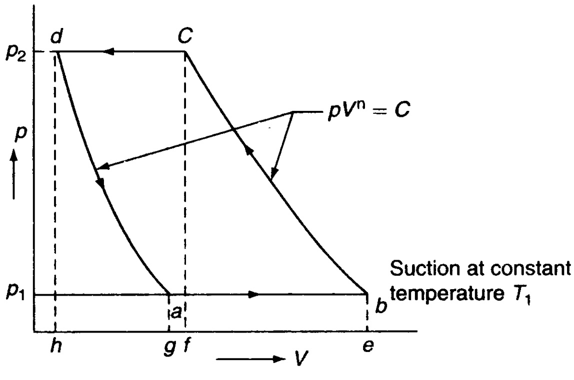


Fig. 19.4 *Indicator diagram*

where $m_b - m_d$ is the difference between the mass of air present at the end of suction and that present at the end of delivery.

Thus the expression Eq. (19.12) is the same as obtained from the steady flow energy equation given in Eq. (19.6) where the clearance volume was neglected. Thus the mass of gas in the clearance volume does not have any effect on the work of compression.

19.4 VOLUMETRIC EFFICIENCY

The amount of air dealt with in a given time by an air compressor is often referred to at free air conditions, i.e. the temperature and pressure of the environment, which may be taken as 15°C and 101.325 kPa, if not mentioned. It is known as free air delivery (FAD). The ratio of the actual volume of gas taken into the cylinder during suction stroke to the piston displacement volume (PD) or the swept volume (V_s) of the piston is called the *volumetric efficiency*, or

$$\eta_{\text{vol}} = \frac{\dot{m}v_1}{PD} = \frac{\dot{m}v_1}{V_s}$$

where $\dot{m}$ is the mass flow rate of the gas and v_1 is the specific volume of the gas at inlet to the compressor. With reference to Fig. 19.5.

$$\eta_{\text{vol}} = \frac{V_2 - V_1}{V_s} = \frac{V_c + V_s - V_1}{V_s} = 1 + \frac{V_c}{V_s} - \frac{V_1}{V_s}$$

$$\text{Let } C = \text{clearance} = \frac{\text{Clearance volume}}{\text{P.D. or } V_s} = \frac{V_c}{V_s}$$

$$\text{Since} \quad p_1 V_1^n = p_2 V_4^n, \therefore V_1 = V_4 \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} = V_c \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$\therefore \quad \eta_{\text{vol}} = 1 + C - \frac{V_c}{V_s}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} = 1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \qquad (19.13)$$

Equation (19.13) is plotted in Fig. 19.6. Since (p_2/p_1) is always greater than unity, it is evident that the volumetric efficiency decreases as the clearance increases and as the pressure ratio increases.

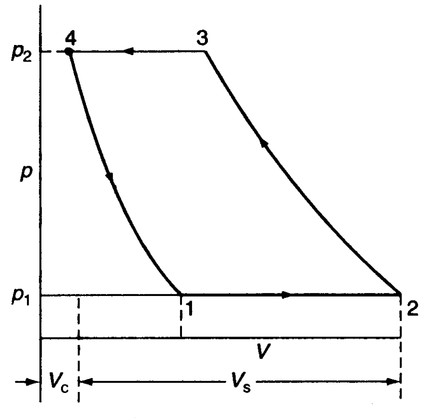


Fig. 19.5 *Volumetric efficiency*

For air compressors, the prescribed inlet conditions are often atmospheric conditions in which the volume of air induced is often called the free air delivery (F.A.D). If the ambient air is at pressure p_a and the temperature T_a, then the volumetric efficiency

$$\eta_{vol} = \frac{V_2 - V_1}{V_2 - V_4} \times \frac{p_1}{p_a} \times \frac{T_a}{T_1}$$

$$= \left[1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\right] \times \frac{p_1}{p_a} \times \frac{T_a}{T_1} \qquad (19.13a)$$

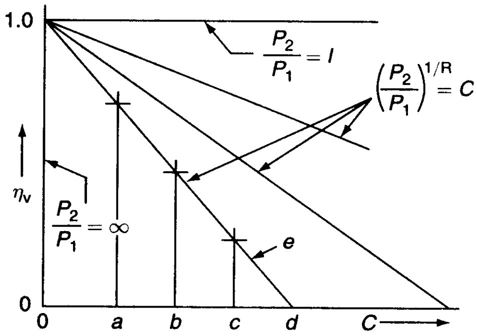


Fig. 19.6 Effect of clearance on volumetric efficiency

In order to get maximum flow capacity, compressors are built with the minimum practical clearance. Sometimes, however, the clearance is deliberately increased $\left(\dot{m} = \frac{V_s n_v}{v_1}\right)$ as a means of controlling the flow through a compressor driven by a constant speed motor. The compressor cylinder of Fig. 19.7 is fitted with a clearance pocket which can be opened at will by a valve. Let us suppose that this machine is operating at conditions corresponding to line e in Fig. 19.6. If the clearance volume is at minimum value a the volumetric efficiency and the flow through the machine are maximum. If the clearance pocket is then opened to increase the clearance to b, the volumetric efficiency and the flow are reduced. By increasing the clearance in steps, as indicated by points c and d, the flow may be reduced in steps to zero. The work per kg of gas compressed is, however, not affected by the clearance volume in an idealized compressor.

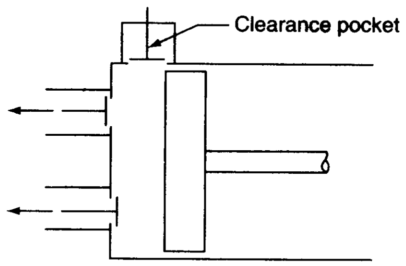


Fig. 19.7 Clearance pocket for capacity control

For a given pressure ratio, η_{vol} is zero when the maximum clearance is

$$C_{max} = \frac{1}{\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1} \qquad (19.14)$$

19.4.1 Effect of Pressure Ratio on Volumetric Efficiency

It is evident from Fig. 19.6 that as the pressure ratio is increased the volumetric efficiency of a compressor of fixed clearance decreases eventually becoming zero. This can also be seen in an indicator diagram, Fig. 19.8. As the discharge pressure is increased from p_{2a} to p_{2b}, the volume V_{1a} taken at pressure p_1 decreases to V_{1b}, and η_{vol} decreases. At some pressure p_{2c} the compression line intersects the line of clearance volume and there is no discharge of gas. An attempt to pump to p_{2c} (or any higher pressure) would result in compression an re-expansion of the same gas repeatedly, with no flow in or out.

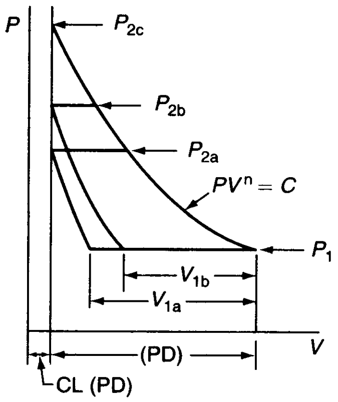


Fig. 19.8 Effect of pressure-ratio on capacity

The maximum pressure ratio attainable with a reciprocating compressor cylinder is then seen to be limited by the clearance, which is given from Eq. (19.15), as

$$\frac{p_{2\max}}{p_1} = \left(1 + \frac{1}{C}\right)^n \tag{19.15}$$

19.5 MULTI-STAGE COMPRESSION

When compressing a gas (or air) to high pressure it is advantageous to do it in stages. The condition for minimum work requires the compression to be isothermal. Since the temperature after compression is given by $T_2 = T_1(p_2/p_1)^{(n-1)/n}$ the delivery temperature, T_2, increases with the pressure ratio. Also the volumetric efficiency as given by Eq. (19.13) decreases as the pressure ratio increases, as mentioned earlier.

The volumetric efficiency can be improved by carrying out the compression in two stages. After the first stage of compression (L.P.) from the state p_1, T_1 to the state p_x, T_x, the fluid is passed into a smaller cylinder (H.P.) in which the gas is compressed to the required final pressure p_2 at temperature T_2 (Fig. 19.9).

The gas after being compressed in the L.P. (low pressure) cylinder ($a - b$) is passed on to an intercooler for getting cooled. After leaving the intercooler the gas enters the H.P. (high pressure) cylinder for further compression (c–d). Figure 19.10 shows the p-V diagram for two-stage compression. Complete or perfect intercooling (b-c) means that the exiting gas from the intercooler at temperature T_x is cooled completely to the original (inlet) temperature T_1.

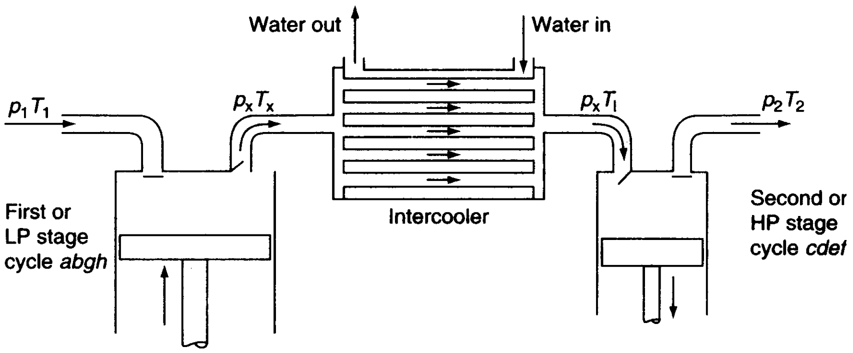


Fig. 19.9 *Plan showing intercooling between compressor stages*

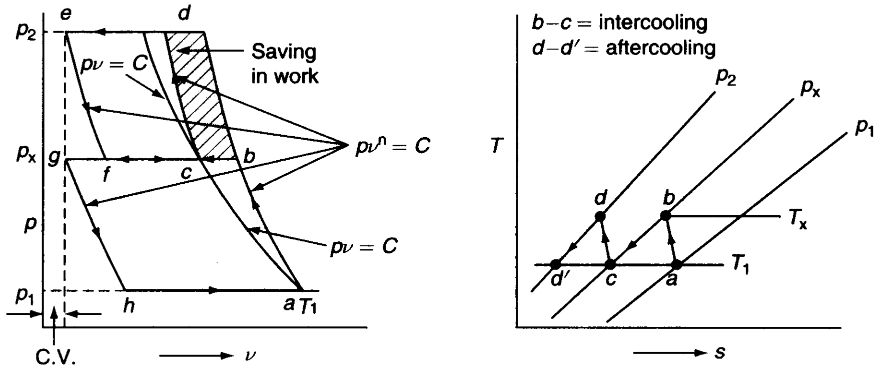


Fig. 19.10 *p-V and T-s diagrams for two-stage compression with perfect intercooling, showing the work saved*

Similarly, perfect aftercooling (d-d') makes the gas, leaving the H.P. compressor, cooled also to the inlet temperature T_1. This aftercooling reduces the volume of the gas leaving, and thus the size of the receiver becomes smaller. The clearance volume in both the cylinders has here been assumed to be the same.

19.5.1 Ideal Intermediate Pressure

The intermediate pressure p_x (Fig. 19.10) has an optimum value for minimum work of compression. The total work per kg of gas is given by

$$W_c = \frac{n}{n-1} RT_1 \left[\left(\frac{p_x}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} RT_1 \left[\left(\frac{p_2}{p_x} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{n}{n-1} RT_1 \left[\left(\frac{p_x}{p_1} \right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p_x} \right)^{\frac{n-1}{n}} - 2 \right] \tag{19.16}$$

Here p_1, T_1 and p_2 are fixed and p_x is the only variable. Differentiating Eq. (19.16) with respect to p_x and making it equal to zero,

$$\frac{dW_c}{dp_x} = \frac{n}{n-1} RT_1 \left[\frac{n-1}{n} \left(\frac{1}{p_1} \right)^{\frac{n-1}{n}} (p_x)^{1-\frac{1}{n}-1} + p_2^{\frac{n-1}{n}} \cdot p_x^{-1+\frac{1}{n}-1} \left(-\frac{n-1}{n} \right) \right] = 0$$

$$\therefore \quad p_x^{-\frac{1}{n}+2-\frac{1}{n}} = (p_2 p_1)^{\frac{n-1}{n}} \quad \text{or,} \quad p_x^{2\left(\frac{n-1}{n}\right)} = (p_1 p_2)^{\frac{n-1}{n}}$$

$$\therefore \quad p_x = \sqrt{p_1 p_2} \tag{19.17}$$

Thus, for minimum work of compression, the intermediate pressure is the geometric mean of the suction and discharge pressures for a two-stage compressor.

From Eq. (19.17)

$$\frac{p_x}{p_1} = \frac{p_2}{p_x} = \left(\frac{p_2}{p_1} \right)^{\frac{1}{2}} \tag{19.18}$$

Pressure ratio in L.P. stage = Pressure ratio in H.P. stage

Also,

$$\frac{T_x}{T_1} = \left(\frac{p_x}{p_1} \right)^{\frac{n-1}{n}} \quad \text{and} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_x} \right)^{\frac{n-1}{n}}$$

$$\therefore \quad T_2 = T_x \tag{19.19}$$

From Fig. 19.6 it is seen that work required in L.P. compressor = Work required in H.P. compressor.

Thus, the intermediate pressure that produces minimum work will also result in equal pressure ratios in the two stages of compression, equal discharge temperatures, and equal work for the two stages.

For two stage-compression, the minimum work, using Eq. (19.16) becomes

$$W_c = \frac{2nRT_1}{n-1} \left[\left(\frac{p_x}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \tag{19.20}$$